Lecture Notes for Intermediate Public Economics

Based on the course content

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Preface

These lecture notes are designed for intermediate-level courses in public economics, based on the curriculum taught for year-3 undergraduates at the School of Public Finance and Taxation, Zhongnan University of Economics and Law. The material bridges the gap between introductory public finance and advanced graduate-level training, providing students with a solid foundation in both theoretical frameworks and empirical applications.

The notes are organized around four core themes: welfare economics and market failures, the economics of externalities and public goods, social insurance, and taxation. Each chapter combines rigorous mathematical derivations with intuitive explanations, real-world examples, and discussions of seminal empirical papers from leading journals. Special attention is given to the fundamental theorems of welfare economics, the Samuelson rule for public goods, the Coase theorem, optimal taxation theory, and the economics of asymmetric information in insurance markets.

I would like to express my gratitude to the instructor of this course, Professor Xiumei Yu. She is amiable and an excellent educator. In this course, she emphasizes not only what economic theory predicts but also how these predictions can be tested empirically. The inclusion of recent studies (I have added hyperlinks in notes) from the *Journal of Public Economics* and other top journals exposes students to cutting-edge research and modern econometric methods, including difference-in-differences, regression discontinuity, and instrumental variables approaches. These lecture notes are a gift for her.

Benefiting from the leisure time after completing graduate school applications, I have had the opportunity to compile these lecture notes. I also believe this is an excellent way to review and prepare for my graduate-level coursework. I personally took this course in the Fall of 2024, and I have also referred to the slides used for this course from 2023 to 2025. Therefore, the notes have evolved over several years of teaching and interactions with students. I hope they serve as a useful resource for both instructors and students in their exploration of public economics.

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Class of 2026
March 10th, 2026

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Chapter 1 Introduction

1.1 Why need to learn Intermediate Public Economics?

After learning *the Principles of Public Finance*, there still are many question like:

- **How the invisible hand plays role in market?**
  The First Theorem of Welfare Economics!
- **How the government provide public goods and correct externalities?**
  The optimal amount of public goods provided.
  Several methods for correcting externalities and their difference.
- **How do you explain the Excess Burden and substitution effect?**
  EV(Equivalent varitation)and **CV**(Compensating variation) to solve excess burden.

1.2 What need to do in this course?

To have a better understanding of this course, you should do:

1.2.1 Finish Homework

Several economic problems that require calculation.

1.2.2 Read literature and presentation

Refer to the list of specific literature given, you need to read and sign up for presentation.

1.2.3 Try to write a model about public economics

Select a topic and try to write and update the model based on the existing models.

1.3 What will be included in course?

- The fundamentals of economic analysis methods
- The theory in Public Revenue
- The theory in Fiscal Expenditure
- The Optimal Tax theory (commodity tax)
- Macro fiscal policy(according to the course progress)

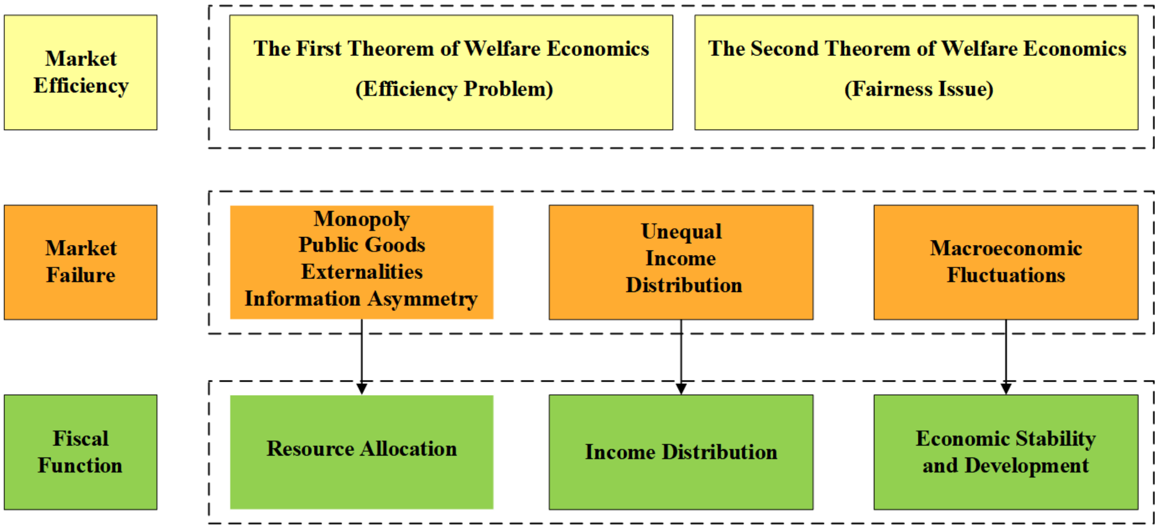
1.4 What can be expected to require after learning?

- Fundamentals of Economic Theory
- Academic Thinking in Economics
- The Skills for Exercise
- Ability to report papers
- High score (depend on yourself!)

Chapter 2 Welfare Theory and Market Failure

2.1 The Structure of this Chapter

This chapter mainly introduces the theories of welfare economics and market failure. From the perspective of the market, the following thinking structure diagram can be obtained:



2.1: The Structrue of this Chapter

2.2 The First Theorem of Welfare Economics

Content: A perfectly competitive market condition will automatically generate a Pareto optimal state of resource allocation.

2.2.1 Market Efficiency

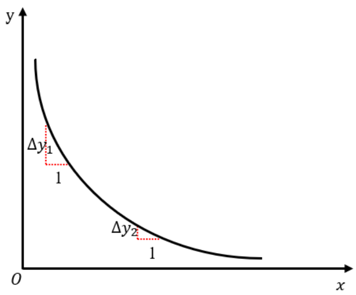
How to define efficiency?

- Pareto improvement: Changes in resource allocation at least improve one person's welfare without causing harm to anyone.
- Pareto optimality: While not making others' situations worse, no one's situation will improve. Thus, resource allocation state without Pareto improvement.

There are some concepts about Pareto optimal allocation in microeconomics, such as:

2.2.2 Indifference Curve

Simply put, **each point on the indifference curve represents the same utility.**



2.2: Indifference curve

- When the amount of x consumed is very small, consumers are willing to give up Δy_1 units of y in order to consume an additional unit of x.
- When consuming a large amount of x, consumers are willing to give up Δy_2 units of y in order to consume an additional unit of x.
- $\Delta y_2 < \Delta y_1$: The indifference curve is convex towards the origin.

Marginal rate of substitution: Slope of indifference curve (absolute value).
As x increases, the marginal substitution rate decreases. Therefore, we can get the fucation :

$$MRS = |\text{the slope of the indifference curve}| = \frac{\text{marginal utility of } x}{\text{marginal utility of } y} = \frac{U_x(x,y)}{U_y(x,y)}$$

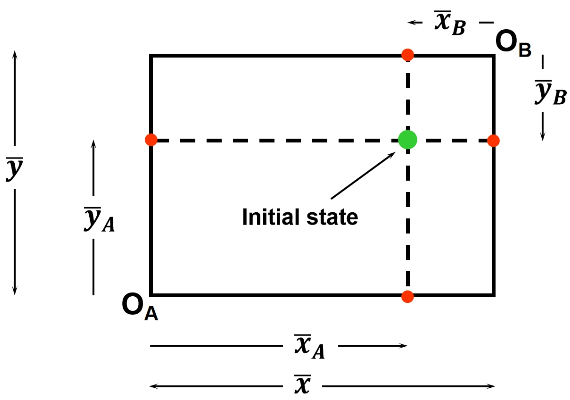
e.g. 1: Assumption: Consuming 1 unit more X increases utility by 2, and consuming 1 unit more Y increases utility by 6.
How many units of Y are consumers willing to give up in order to consume an extra unit of X?

Marginal substitution rate=marginal utility of X/marginal utility of Y=2/6

2.2.3 Edgeworth Box

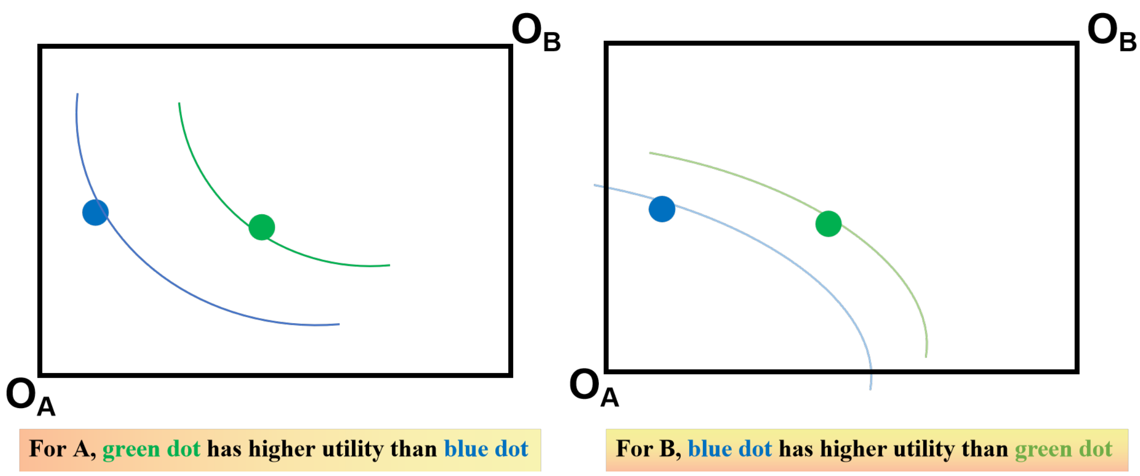
A tool to illustrate the potential outcomes of trade between two individuals or parties, given their preferences, endowments, and consumption of two goods. It is a square box that represents all the possible allocations of two goods between two people. The horizontal and vertical axes of the box measure the quantity of the two goods, one for each person.

A point within the box represents the initial endowments of the individuals, and indifference curves are drawn to show combinations of the goods that give the individuals equal levels of satisfaction.



2.3: Initial state

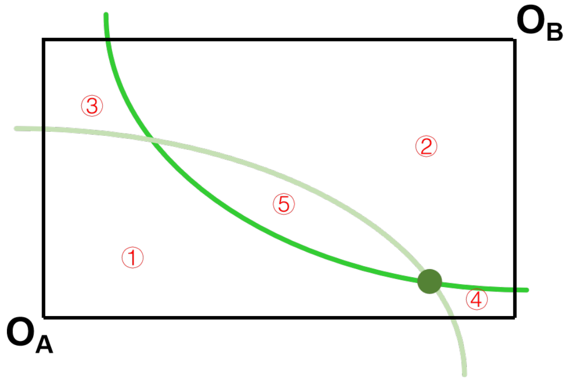
For two independent individuals, A and B, representing their indifference curves in the Edgeworth Box as Figure 2.4. We can compare their utility respectively.



2.4: Comparing the utility of two dots

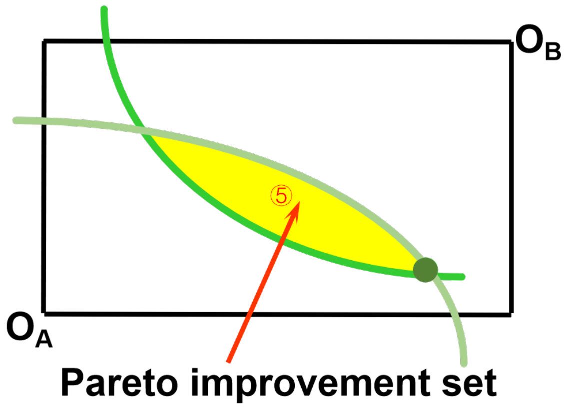
Additionally, we can darw their indifference curves in one figure. We can consider **whether there is a Pareto improvement on a certain optional configuration**. If so, which of the regions (1 to 5) in the figure represents higher utility for both individuals?

To solve this question, we can compare the utility represented by the indifference curve in each region. The regions which have the higher utility are what we need.



2.5: Optional configuration

We first select A for analysis (B is symmetrical). For A, regions 2 and 5 are areas where its utility can be improved, as the indifference curves in these regions represent higher levels of utility. And the regions of 1,3,4 have lower utility. Similarly, 1 and 5 are the results of B's selection. Therefore, we can derive **the Pareto improvement set**.

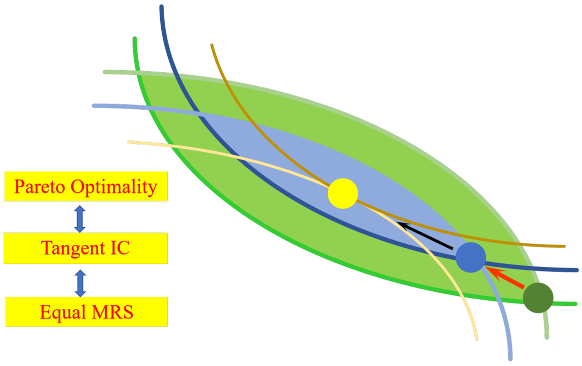


2.6: Pareto improvement set

Pareto Optimality:
If both individuals continuously improve their respective utility at the same time, the final Pareto improvement set will become an empty set, which means reaching Pareto optimality and unable to continue improving.

Focusing on local characteristics, it can be observed that the indifference curve is precisely tangent when reaching Pareto optimality. At the same time, combined with the formula for marginal substitution rate in the previous section, it is not difficult to conclude that when Pareto optimal, the two marginal substitution rates are also equal.

We assume that if the marginal rates of substitution are not equal.Then for two people who can freely exchange goods, mutual exchange will make them both better. This is equivalent to performing Pareto improvement and not achieving Pareto optimality



2.7: Pareto Optimality

We can give an example from daily life to better understand the situation of inconsistent marginal replacement rates.

e.g. 2: A believes that 1 burger is worth 6 breads.
B believes that 1 burger is worth 3 breads.
Can be exchanged, such as: A takes 4 breads and exchanges them with B for 1 burger.

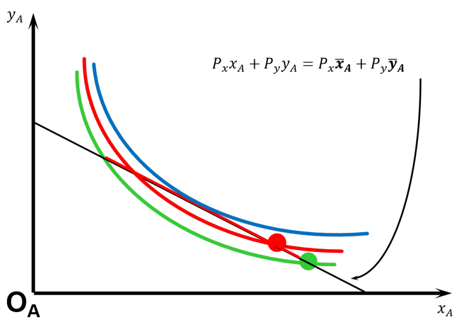
A gains 1 burger (equivalent to 6 bread), loses 4 breads, and earns a net profit of 2 breads.
B gets 4 breads, loses 1 burger (equivalent to 3 breads), equivalent to a net profit of 1 bread.
Both of them are better, that is, they have achieved Pareto improvement.

2.2.4 Perfect Competition Market

In a perfectly competitive market, both A and B are **price takers**.
That is, given the prices P_x and P_y,they choose the appropriate consumption combination $\{x_A, y_A\}$, $\{x_B, y_B\}$ to maximize utility.
The problem of maximizing the utility of A:

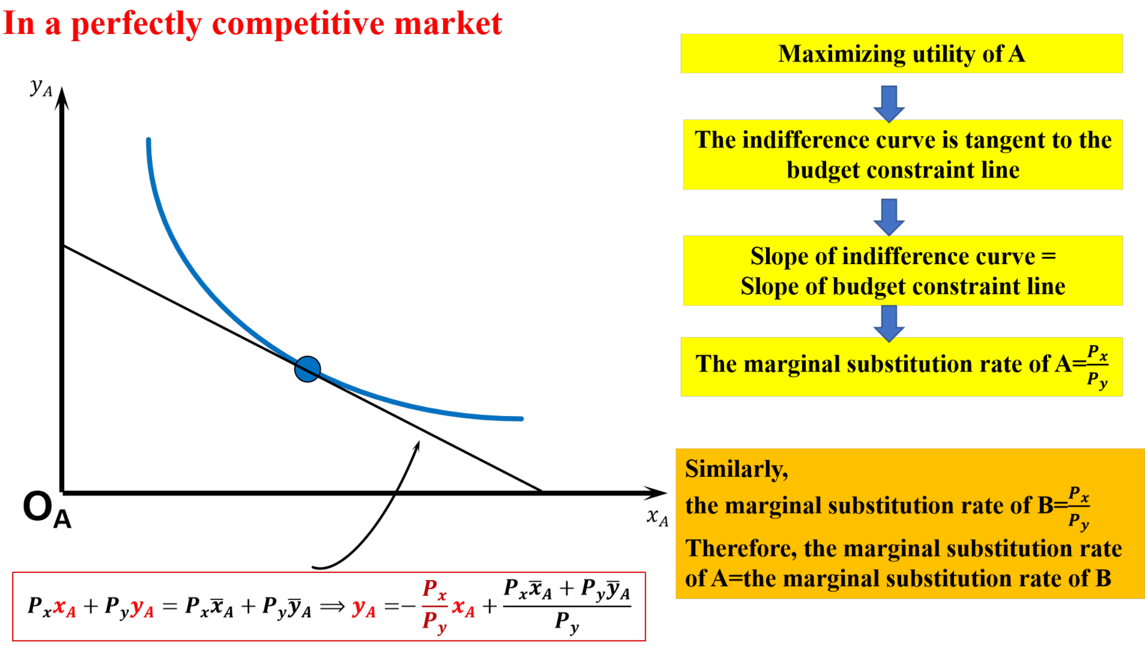
$$\max u(x_A, y_A)$$
$$s.t.\quad P_x x_A + P_y y_A = P_x \bar{x_A} + P_y \bar{y_A}$$

where the left side represents expenses, and the right side represents income.



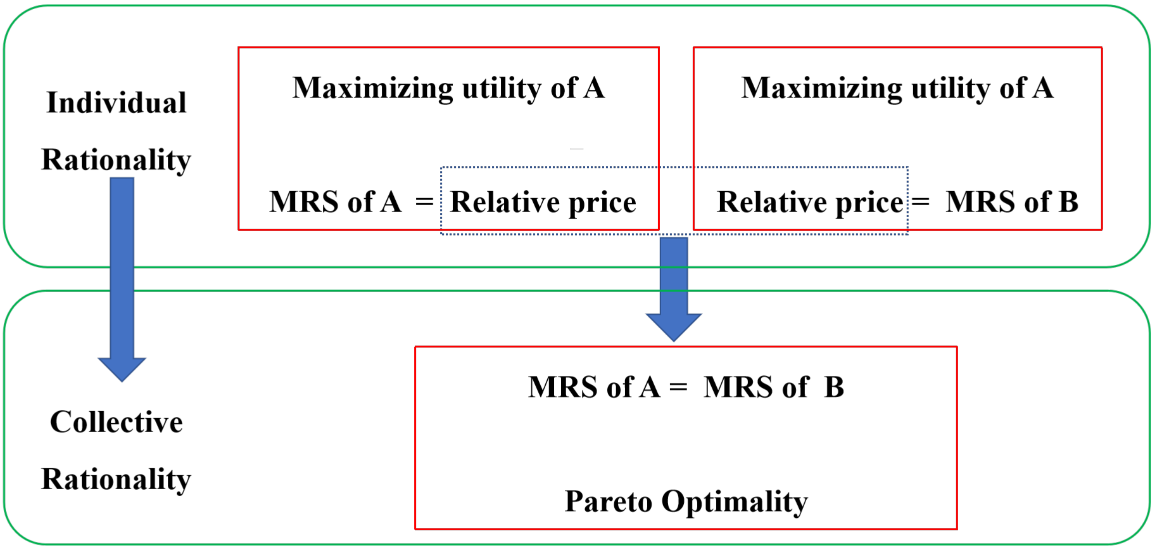
2.8: Maximizing the utility for A

Find the position where the indifference curve is tangent to the budget constraint line, and perform algebraic solution at that location.



2.9: Tangent situation

To summarize briefly, we can find the relationship between the above concepts and sort them out as the figure below:



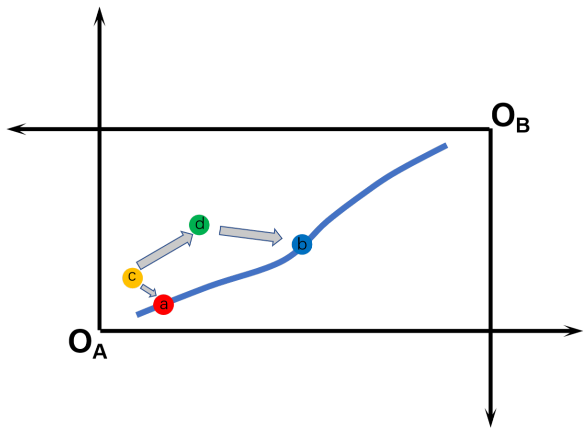
2.10: Relationship

It can be inferred from Pareto optimal allocation and perfectly competitive markets that **resource allocation in perfectly competitive markets is Pareto optimal.**

2.3 The Second Theorem of Welfare Economics

A and B are both on the contract curve and Pareto optimal, which one is fairer? How to change the configuration result from a to b:

- Change initial configuration (Fairness)
- Make the market work (Efficiency)



2.11: Changing the configuration

Content: The Second Fundamental Theorem of Welfare Economics states that if every consumer has convex preferences and every firm has a convex production set then any Pareto-efficient allocation can be decentralized as a competitive equilibrium.

The second theorem of welfare economics has certain advantages over first theorem of welfare economics. It explains that if all consumers have convex preferences and all firms have convex production possibility sets then Pareto efficient allocation can be achieved. The equilibrium of a complete set of competitive markets are suitable for redistribution of initial endowments.

Analysis:

- The initial allocation of resources determines the final allocation of resources.
- Equity and efficiency can be considered separately.
  - The government adjusts the initial allocation of resources to achieve equity.
  - The market's "invisible hand" achieves efficiency.

2.4 How to solve the problem of maximizing utility?

To solve the utility maximization problem, we need obey such a rule : If a consumer wants to maximize total utility, for every dollar that they spend, they should spend it on the item which yields the greatest marginal utility per dollar of expenditure.

Here is an example with step-by-step mathematics.

$$\max_{x_A, y_A} u(x_A, y_A) = x_A^\alpha y_A^\beta$$

s.t.

$$P_x x_A + P_y y_A = P_x \bar{x}_A + P_y \bar{y}_A \equiv I_A$$

From the budget constraint:

$$y_A = \frac{I_A - P_x x_A}{P_y}$$

Substituting into the utility function:

$$u(x_A, y_A) = x_A^\alpha \left(\frac{I_A - P_x x_A}{P_y} \right)^\beta$$

Taking the first-order derivative with respect to x_A:

$$\alpha x_A^{\alpha-1} \left(\frac{I_A - P_x x_A}{P_y} \right)^\beta - x_A^\alpha \beta \left(\frac{I_A - P_x x_A}{P_y} \right)^{\beta-1} \frac{P_x}{P_y} = 0$$

Simplifying:

$$\alpha \left(\frac{I_A - P_x x_A}{P_y} \right) = \beta \frac{P_x}{P_y} x_A$$

That is:

$$P_x X_A = \frac{\alpha}{\alpha + \beta} I_A$$

Conclusion: When the utility function takes the form $x_A^\alpha y_A^\beta$, the consumer spends a fraction $\frac{\alpha}{\alpha+\beta}$ of their income on x_A and $\frac{\beta}{\alpha+\beta}$ on y_A. (This result can be used directly in our coursework or exams, but the consumer utility maximization problem must be stated.)

Algebraic Version of the First Fundamental Theorem of Welfare Economics: Market Solution

- Given P_x, P_y, A solves the utility maximization problem to obtain:

$$x_A = \frac{\alpha}{\alpha + \beta} \frac{P_x \bar{x}_A + P_y \bar{y}_A}{P_x}, \quad y_A = \frac{\beta}{\alpha + \beta} \frac{P_x \bar{x}_A + P_y \bar{y}_A}{P_y} \tag{1}$$

- Given P_x, P_y, B solves the utility maximization problem to obtain:

$$x_B = \frac{\alpha}{\alpha + \beta} \frac{P_x \bar{x}_B + P_y \bar{y}_B}{P_x}, \quad y_B = \frac{\beta}{\alpha + \beta} \frac{P_x \bar{x}_B + P_y \bar{y}_B}{P_y} \tag{2}$$

- Solve for prices using the market equilibrium condition: $x_A + x_B = \bar{x}_A + \bar{x}_B$, i.e.,

$$\frac{\alpha}{\alpha+\beta}\frac{P_x\bar{x}_A + P_y\bar{y}_A}{P_x} + \frac{\alpha}{\alpha+\beta}\frac{P_x\bar{x}_B + P_y\bar{y}_B}{P_x} = \bar{x}_A + \bar{x}_B$$

The relative price can be solved as:

$$\frac{P_y}{P_x} = \frac{\beta}{\alpha}\frac{\bar{x}}{\bar{y}}$$

Substituting this relative price back into equations (1) and (2) yields the allocation under perfect competition.

Algebraic Version of the First Fundamental Theorem of Welfare Economics: Pareto Optimal Solution

The Pareto optimal solution satisfies:

$$\max_{x_A, y_A, x_B, y_B} \alpha_1 u(x_A, y_A) + \alpha_2 u(x_B, y_B)$$

$$s.t. \quad x_A + x_B = \bar{x}, \quad y_A + y_B = \bar{y}$$

The Lagrangian equation is:

$$\mathcal{L} = \alpha_1 u(x_A, y_A) + \alpha_2 u(x_B, y_B) + \lambda_1(\bar{x} - x_A - x_B) + \lambda_2(\bar{y} - y_A - y_B)$$

First-order conditions:

$$\begin{aligned}
(x_A): &\quad \alpha_1 MU_A^x = \lambda_1 \\
(y_A): &\quad \alpha_1 MU_A^y = \lambda_2 \\
(x_B): &\quad \alpha_2 MU_B^x = \lambda_1 \\
(y_B): &\quad \alpha_2 MU_B^y = \lambda_2
\end{aligned}$$

Therefore, the marginal rates of substitution for both individuals are equal:

$$\frac{MU_A^x}{MU_A^y} = \frac{MU_B^x}{MU_B^y} = \frac{\lambda_1}{\lambda_2}$$

Marginal Rate of Substitution =

$$\frac{MU_x}{MU_y} = \frac{\alpha x^{\alpha-1} y^{\beta}}{\beta x^{\alpha} y^{\beta-1}} = \frac{\alpha y}{\beta x}$$

At the Pareto optimum, the marginal rates of substitution for A and B are equal, i.e.:

$$\frac{\alpha y_A}{\beta x_A} = \frac{\alpha y_B}{\beta x_B}$$

Given the resource constraints $x_A + x_B = \bar{x}$, $y_A + y_B = \bar{y}$, the contract curve is:

$$y_A = \frac{\bar{y}}{\bar{x}} x_A$$

2.5 Market Efficiency and Market Failure

A perfectly competitive market operates under specific conditions to achieve efficiency: both consumers and producers are price takers. There is a large number of participants in the market, products are homogeneous, resources are fully mobile, and all agents possess complete information.

However, when these conditions are not met, market failures occur. Common sources of market failure include the presence of public goods, externalities, information asymmetry, monopoly power, unequal income distribution, and macroeconomic instability.

2.6 Extension: Generalization of the First Fundamental Theorem of Welfare Economics

Consider a perfectly competitive pure exchange economy with m representative participants and n types of goods. The initial endowment of the i-th participant is given by $(\bar{x}_1^i, \bar{x}_2^i, \ldots, \bar{x}_n^i)$. We aim to prove that in such an economy, the perfectly competitive market equilibrium is always Pareto optimal.

Notation

Let

$$\bar{x}_j = \bar{x}_j^1 + \bar{x}_j^2 + \cdots + \bar{x}_j^m$$

denote the total amount of good j available in the economy. Let x_j^i denote the quantity of good j consumed by consumer i.

Pareto Optimal Solution

The Pareto optimal allocation is found by solving the following optimization problem:

$$\max \ \alpha_1 U(x_1^1, x_2^1, \ldots, x_n^1) + \alpha_2 U(x_1^2, x_2^2, \ldots, x_n^2) + \cdots + \alpha_m U(x_1^m, x_2^m, \ldots, x_n^m)$$

subject to the resource constraints:

$$\begin{aligned}
x_1^1 + x_1^2 + \cdots + x_1^m &= \bar{x}_1 \\
x_2^1 + x_2^2 + \cdots + x_2^m &= \bar{x}_2 \\
&\vdots \\
x_n^1 + x_n^2 + \cdots + x_n^m &= \bar{x}_n
\end{aligned}$$

The Lagrangian is:

$$\begin{aligned}
\mathcal{L} =& \alpha_1 U(x_1^1, \ldots, x_n^1) + \alpha_2 U(x_1^2, \ldots, x_n^2) + \cdots + \alpha_m U(x_1^m, \ldots, x_n^m) \\
&+ \lambda_1(\bar{x}_1 - x_1^1 - x_1^2 - \cdots - x_1^m) \\
&+ \lambda_2(\bar{x}_2 - x_2^1 - x_2^2 - \cdots - x_2^m) \\
&+ \cdots \\
&+ \lambda_n(\bar{x}_n - x_n^1 - x_n^2 - \cdots - x_n^m)
\end{aligned}$$

First-Order Conditions

For consumer 1:

$$\alpha_1 MU_1^1 = \lambda_1, \quad \alpha_1 MU_2^1 = \lambda_2, \quad \ldots, \quad \alpha_1 MU_n^1 = \lambda_n$$

For consumer i:

$$\alpha_i MU_1^i = \lambda_1, \quad \alpha_i MU_2^i = \lambda_2, \quad \ldots, \quad \alpha_i MU_n^i = \lambda_n$$

Thus, the condition for Pareto optimality is:

$$MRS_{j1}^1 = MRS_{j1}^2 = \cdots = MRS_{j1}^m, \quad j = 2, 3, \ldots, n$$

That is, the marginal rates of substitution between any two goods are equal across all consumers.

Competitive Market Solution

Given prices, the utility maximization problem for consumer i is:

$$\max U(x_1^i, x_2^i, \ldots, x_n^i)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} P_j x_j^i = I^i$$

The Lagrangian is:

$$\mathcal{L} = U(x_1^i, \ldots, x_n^i) + \theta_i \left(I^i - \sum_{j=1}^{n} P_j x_j^i \right)$$

First-order conditions:

$$MU_j^i = \theta_i P_j, \quad j = 1, 2, \ldots, n$$

Therefore, in the market equilibrium:

$$MRS_{j1} = \frac{MU_j^i}{MU_1^i} = \frac{P_j}{P_1}, \quad j = 2, 3, \ldots, n$$

Since all consumers face the same prices, we have:

$$MRS_{j1}^1 = MRS_{j1}^2 = \cdots = MRS_{j1}^m = \frac{P_j}{P_1}$$

This exactly matches the Pareto optimality condition. Hence, the competitive market equilibrium is Pareto optimal.

Chapter 3 Externality

This chapter classifies externalities, introduces the Coase Theorem, which shows how private bargaining can achieve efficiency under zero transaction costs, and examines corrective policies. Finally, we reviews a JPE study that empirically estimates the welfare implications of a major environmental externality.

3.1 Definition and Connotation

Externality refers to the impact of a party's actions on a third party who is not directly involved in the economic transaction. Also, we can describe as **the inconsistency between private costs and social costs or between private benefits and social benefits**.

3.1.1 What is the externality?

To begin with, we assume two situation and find which externality need to correct.

Definition 3.1.1: Pecuniary Externality

Assuming an increase in gasoline prices in the market, individual A contemplates the purchase of a new energy electric vehicle. However, a concomitant rise in demand for such vehicles, driven by similar considerations among other market participants, leads to an escalation in their prices. Consequently, the purchasing behavior of others imposes an economic detriment on A.

Actucally, this externality is transmitted through price signals within the market mechanism, and thus is not an issue.

Definition 3.1.2: Non-pecuniary externality

Another type of externality refers to the direct impact of an individual's actions on another person's utility or production. For example, the exhaust gases emitted by a factory can directly affect the health of nearby residents.

Unlike the former, this type of externality is not transmitted through market mechanisms, or rather, the market mechanism is absent.

Aboustely, the non-pecuniary externality is what we will focus on. We can define that **an externality is a cost or benefit that is caused by one party but financially incurred or received by another**. Externalities can be negative or positive. A negative externality is the indirect imposition of a cost by one party onto another. A positive externality, on the other hand, is when one party receives an indirect benefit as a result of actions taken by another.

3.1.2 Some classifications

Externalities can be broken into two different categories. First, externalities can be measured as good or bad as the side effects may enhance or be detrimental to an external party. These are referred to as positive or negative externalities. Second, externalities can be defined by how they are created. Most often, these are defined as a production or consumption externality.

Negative Externalities

Most externalities are negative. Pollution is a well-known negative externality. A corporation may decide to cut costs and increase profits by implementing new operations that are more harmful to the environment. The corporation realizes costs in the form of expanding operations but also generates returns that are higher than the costs.

However, the externality also increases the aggregate cost to the economy and society making it a negative externality. Externalities are negative when the social costs outweigh the private costs.

Positive Externalities

Some externalities are positive. Positive externalities occur when there is a positive gain on both the private level and social level. Research and development (RD) conducted by a company can be a positive externality. RD increases the private profits of a company but also has the added benefit of increasing the general level of knowledge within a society.

Similarly, the emphasis on education is also a positive externality. Investment in education leads to a smarter and more intelligent workforce. Companies benefit from hiring employees who are educated because they are knowledgeable. This benefits employers because a better-educated workforce requires less investment in employee training and development costs.

Production Externalities

A production externality is an instance where an industrial operation has a side effect. This is often the type of externality used as example, as it is easy to envision an environmental catastrophe caused by improperly stored chemicals by a chemical company. Because of how the company produced its goods or protected its waste, an externality occurred.

Consumption Externalities

Externalities may also occur based on when or how a consumer base utilizes resources. Consider the example of how you get to work. Those who choose to drive are creating a pollution externality by driving their own car. Those who choose to take public transit or walk are not causing the same externality. Instead of a side effect occurring because something is being produced, an externality is caused because of an item being consumed.

3.2 An Example with Model to Explain Externalities

3.2.1 Set up

We build the model in both consumer and firm as follow:

Consider two goods:

- x represents the number of cars and y represents the consumption of all other goods.

- Let y be the numeraire, meaning the price of y is 1, and the price of x is p.

For a representative consumer with 1 unit of income, the utility function is given by:

$$u(x) + y - dX$$

This is **a quasi-linear utility functio**n where:

- d is the disutility per car produced due to pollution, reducing the consumer's utility by d units.
- X is the total car production in society.

  At equilibrium, $x = X$, but the consumer cannot choose X individually; X is determined collectively by all consumers in society.
- The consumer's income is I.

For the firm of production:

- The cost of producing cars is $c(X)$.

In this context, the consumer's utility is affected by the total production of cars in society, which they do not control directly. This introduces an externality into the consumer's utility function, as the pollution generated by car production affects their well-being but is not reflected in the price they pay for the car.

3.2.2 Perfect competition market solution

We formulate the above setup into the model form of economics as follow.

Consumer Utility Maximization:

$$\max U(x) + y - dX$$
$$\text{s.t. } px + y = I$$

That is, maximize: $u(x) + I - px - dX$ (Certainly, you can also establish a Lagrange function for solving this problem. However, due to the simplicity of the functional form, we just use the substitution method.)

First-order condition:

$$u'(x) = p$$

Producer Profit Maximization:

$$\max pX - c(X)$$

First-order condition:

$$p = c'(X)$$

Market Equilibrium Solution:

$$u'(x) = c'(x)$$

Note: The negative externality of pollution is not considered!

3.2.3 Social optimum solution

The social optimum solution can be translated into the following economic model:

$$\max u(x) + y - dX$$
$$\text{s.t. } x = X, c(x) + y = I$$

That is, maximize:

$$u(x) + I - c(x) - dx$$

First-order condition:

$$u'(x) = c'(x) + d$$

This represents the condition for achieving the social optimum where the marginal utility of the last unit of the good consumed equals the sum of the marginal cost of production and the marginal disutility caused by the externality (in this case, pollution).

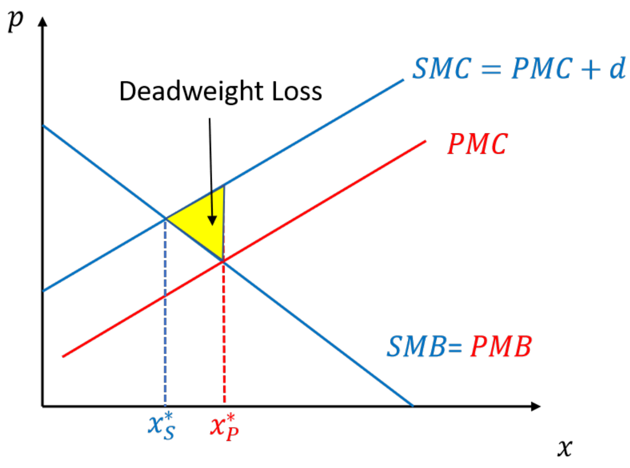
We compare two solution in a table like this:

	Marginal Benefit	Marginal Cost
Market solution	$PMB = u'(x)$	$PMC = c'(x)$
Social solution	$SMB = u'(x)$	$SMC = c'(x) + d$

3.3 Correct Enternalities

3.3.1 Why we need?

Due to the ***Deadweight Loss***!
It refers to the decrease in economic efficiency that can occur when a market is not operating at its optimal level, typically due to some form of market failure such as monopoly, tariffs, quotas, taxes, or other distortions.. In our case, the PMC is not equal to SMC due to the air pollution.



3.1: Deadweight Loss

3.3.2 Market Failure under Externalities

Why does market failure occur?
Private decision-makers do not account for the social cost of pollution when making choices.
The socially optimal level of pollution

- It is determined where the social marginal benefit equals the social marginal cost:

$$PMB = PMC + MD$$

- This requires knowledge of PMB, PMC, and MD (marginal damage).
- Since there is no market for pollution, MD is difficult to measure.
- Note: The socially optimal level of pollution is not necessarily zero.

We have several policy instruments to correct externalities as follow.

- Clarify property rights (Coasian solution)
- Price-based approaches
  - Taxes or subsidies (Pigouvian taxation)
- Quantity-based approaches
  - Command and control (government regulation)
  - Cap and trade (emissions trading)

3.4 Coase Theorem

3.4.1 Defination and Example

In this section, we will introduce the Coase theorem and its methods for solving externalities through examples.

Definition 3.4.1: Coase Theorem

If the transaction costs in a market are zero, regardless of the initial allocation of property rights, the market mechanism will automatically lead to a Pareto optimal allocation of resources.

e.g. 3: A real-life example

A: Lover of Luosifen (rice noodles with a strong flavor). The utility of eating Luosifen is 10.
B: Dislikes the smell of Luosifen. The utility of clean air is 20.

Social Optimum:

- If A eats Luosifen: A's utility = 10, B's utility = 0, total utility = 10.
- If A does not eat Luosifen: A's utility = 0, B's utility = 20, total utility = 20.
- Therefore, **not eating Luosifen** is the socially optimal outcome.

Case 1: A has the right to eat Luosifen

- B can pay A 15 to refrain from eating Luosifen.
- Outcome: A's utility = 15, B's utility = 5. Total utility = 20.

Case 2: B has the right to clean air

- A and B cannot reach a mutually beneficial agreement (A cannot afford to compensate B).
- Outcome: A does not eat Luosifen. A's utility = 0, B's utility = 20. Total utility = 20.

In both cases, the final allocation is Pareto optimal (total utility = 20), regardless of who initially holds the property right.

3.4.2 A Mathematical Example

We now turn to an example with mathematical derivations: fuel-powered cars and air quality. Consider a simplified economy with two agents: a car manufacturer and a consumer. The production of cars generates pollution that negatively affects the consumer's utility. We analyze two different assignments of property rights over clean air and show that both lead to the same socially optimal outcome when bargaining is possible.

Case 1: Clean Air Rights Belong to the Consumer

- For each unit of car produced, the manufacturer must pay the consumer a pollution fee d.
- **Firm's profit maximization**:

$$\max_{x} \; px - C(x) - dx$$

$$p = C'(x) + d$$

- **Consumer's utility maximization**: The consumer's utility depends on car consumption x, other goods y, and the total pollution dX (where X is the total cars produced).

$$\max_{x,y} \; u(x) + y - dX$$

$$\text{s.t.} \quad px + y = I + dX$$

  Substituting y from the budget constraint:

$$u(x) + I - px$$

  First-order condition:

$$p = u'(x)$$

- **Equilibrium condition**:

$$p = u'(x) = C'(x) + d$$

Case 2: Clean Air Rights Belong to the Firm

- The consumer is willing to pay d per unit to induce the firm to reduce production by one car.
- **Firm's profit maximization**: The firm receives a payment of d for each unit it reduces from a baseline level X_p^*.

$$\max_x \; px - C(x) + d(X_p^* - x)$$

First-order condition:

$$p = C'(x) + d = SMC$$

- **Consumer's utility maximization**: The consumer's budget constraint now reflects the payment to the firm for pollution reduction.

$$\max_{x,y} \; u(x) + y - dX$$
$$\text{s.t.} \quad px + y = I - d(X_p^* - X)$$

Substituting y:

$$u(x) + I - dX_p^* - px$$

First-order condition:

$$p = u'(x)$$

- **Equilibrium condition**:

$$p = u'(x) = C'(x) + d$$

Conclusion

In both cases, the equilibrium condition is identical:

$$p = u'(x) = C'(x) + d$$

This demonstrates the **Coase Theorem** in action: regardless of whether the property right to clean air is assigned to the consumer or the firm, the market achieves the same efficient outcome, provided that transaction costs are zero and bargaining is possible.

3.5 Solution: Taxes or Subsidies

In last example, a tax of d per unit of car is imposed on the producer. Then, we hold **Producer's profit maximization**:

$$\max_x \; px - C(x) - dx$$

First-order condition:

$$p = C'(x) + d = SMC$$

In general, the tax should be set equal to the difference between the social marginal cost and the private marginal cost.
Then the next question comes: **Coasian Solution or Pigouvian Tax?**
Both approaches require knowledge of the marginal damage (MD).

- Under clearly defined property rights, if MD is unknown, information asymmetry may lead to market failure.
- When a tax is imposed, the tax rate should equal MD.

Challenges of the Coasian Solution:

- Bargaining costs can be high, especially when a firm needs to negotiate with each individual consumer.
- It is more suitable for **thin markets** with a small number of participants.

Potential Problems with Pigouvian Taxes:

- May lead to a **local optimum** rather than a global optimum (Baumol, 1972).
  - Example: A factory pollutes a river. Possible solutions include reducing emissions or relocating the factory.
  - Under clearly defined property rights, the least costly option will be chosen.
  - With a Pigouvian tax, the factory may choose to reduce emissions, which could attract more people to move near the river, thereby increasing the marginal damage.
- Pigouvian taxes are more suitable for **thick markets** with many participants.

3.6 Government Regulation (Command and Control)

Imagine a city plagued by smog. The government steps in and tells every factory: "You must reduce your emissions by exactly 30%." This is straightforward, but is it efficient? This is the essence of **command-and-control regulation**—one of the oldest and most direct approaches to correcting market failures caused by externalities.

The government directly sets a pollution limit, e.g., the car factory's emissions cannot exceed x^*. In some cases, firms are required to adopt specific abatement technologies.

- **Advantages**
  - Easy to implement and enforce.
  - Can quickly achieve a targeted reduction in pollution.
- **Disadvantages**
  - Provides little incentive for firms to innovate and develop greener technologies beyond the mandated standard.
  - Ignores heterogeneity across firms: some firms have high abatement costs, while others can reduce pollution at low cost. A uniform standard is therefore inefficient.

3.7 Emissions Trading (Cap and Trade)

Of course, an alternative approach is to directly **regulate emissions** through command-and-control. Market-based instruments like cap and trade often offer greater flexibility and efficiency.

After setting a total emission cap, the government allows firms to trade pollution permits.

- High-abatement-cost firms can buy permits from low-abatement-cost firms.
- Advantages Compared to Command-and-Control Regulation
  - **Recognizes firm heterogeneity**: Firms with low abatement costs will reduce emissions more and sell their excess permits; firms with high abatement costs can buy permits instead of reducing emissions at prohibitively high cost. This ensures that the overall pollution reduction is achieved at the lowest possible cost.
  - **Encourages innovation**: Firms have a continuous incentive to develop greener technologies, as they can profit by selling their unused permits.

3.8 Pigouvian Tax vs. Cap and Trade: Price vs. Quantity

The policymaker trying to reduce pollution has two powerful tools : either **set a price on pollution** (Pigouvian tax) and let the market determine the quantity, or **set a quantity limit** (cap and trade) and let the market determine the price. Which one is better?

- **Pigouvian Tax**: Price-based instrument
- **Cap and Trade**: Quantity-based instrument
- **Price vs. Quantity**
  - In our earlier example of car production, with perfect information, both instruments achieved the same efficient outcome.
  - However, when there is uncertainty about the marginal cost or marginal benefit of pollution reduction, the two approaches may lead to different outcomes. (See Weitzman, *Review of Economic Studies*, 1974)

The answer, as Martin Weitzman showed in his classic 1974 paper, depends on uncertainty. Weitzman's key insight: If the marginal benefit curve is relatively flat (i.e., the social damage from pollution is insensitive to the quantity), a price instrument (tax) is preferable. If the marginal cost curve is relatively flat, a quantity instrument (cap) is preferable. The choice hinges on which uncertainty—cost or benefit—matters more for social welfare.

3.9 How Large Is the Externality? Measuring the Cost of Pollution

When a factory emits pollution, it imposes a cost on society. But how large is that cost? Unlike apples or cars, pollution does not have a market price. Economists have developed creative ways to infer its value from observable behavior. Two prominent approaches are:

- **Hedonic Price Method**: Uses indirect market prices to infer the value of environmental goods. For example, how much does air pollution affect housing prices? (Chay and Greenstone, *JPE* 2005)
- **Contingent Valuation**: Uses surveys to ask people how much they would be willing to pay for cleaner air.

Chay and Greenstone, JPE 2005

Research Question: By how much does a one-unit increase in air pollution reduce housing prices?

- **Linear Regression**:

$$\text{HousingPrice}_{ct} = \alpha \text{Pollution}_{ct} + X_{ct} + \delta \lambda_c + \gamma_t + \varepsilon_{ct}$$

- **Two-Period Data (1970 and 1980)**: With two periods, fixed effects are equivalent to first differences.

$$\Delta \text{HousingPrice}_c = \alpha \Delta \text{Pollution}_c + \delta \Delta X_c + \mu_c$$

  where $\Delta X_c = X_{c,1980} - X_{c,1970}$.

- **Potential Biases in OLS**:
  - **Omitted Variable Bias**:
    * Without city fixed effects, high-pollution areas tend to be urban, and urban areas have higher housing prices. This biases α upward (less negative).
    * Economic booms increase both pollution and housing prices, also biasing α upward.
  - **Self-Selection Bias**:
    * People who are less sensitive to pollution may choose to live in high-pollution areas, making housing prices less responsive to pollution (biasing α toward zero).

To address these biases, Chay and Greenstone use an instrumental variable that affects pollution but not housing prices directly.

Instrumental Variables Approach

For those unfamiliar with instrumental variables (IV), let us briefly review the core idea from econometrics.

Definition 3.9.1: Instrumental Variable: An Analogy to Natural Science Experiments

An instrumental variable is a third variable introduced into regression analysis that is correlated with the predictor variable, but uncorrelated with the response variable. By using this variable, it becomes possible to estimate the true causal effect that some predictor variable has on a response variable

Imagine we want to study the causal effect of **pollution on housing prices**. This is similar to a natural science question like: **Does disease X affect sleep quality?**

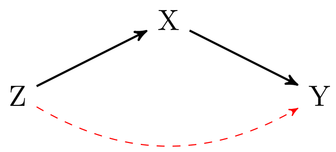
If we simply look at observational data and regress sleep quality on disease X, we may find a correlation. But correlation is not causation. Why?

- **Omitted variable bias**: Pessimistic people may be more likely to contract disease X and also tend to have poor sleep quality.

- **Reverse causality**: People with chronic poor sleep may be more susceptible to disease X.

In both cases, the estimated relationship between X and Y is biased. We need a way to isolate the causal channel.

The Role of an Instrumental Variable (Z)



An instrumental variable Z must satisfy two key conditions:

1. **Relevance**: Z must affect X.
    - In a drug trial, Z (the treatment) must be effective against disease X—it cannot be a placebo like flour or water.
2. **Exclusion restriction**: Z must affect Y only through X.
    - Z cannot directly affect Y (e.g., if the drug causes insomnia or drowsiness as a side effect, it is not a good instrument).
    - Z cannot affect Y through any other channel (e.g., if the drug causes headaches, which then affect sleep quality, the exclusion restriction is violated).

Back to Chay and Greenstone, JPE 2005

- **Instrument**: The 1970 U.S. Clean Air Act Amendments.
    - The Act set standards for five major air pollutants.
    - Counties that failed to meet the standards were designated as "nonattainment" areas and were required to take measures to improve air quality.
    - Nonattainment status is a strong predictor of changes in pollution, but should not directly affect housing prices except through pollution.
- **First Stage**:
$$\Delta \text{Pollution}_c = \beta_1 Z_c + \beta_2 \Delta X_c + \varepsilon_c$$
where Z_c is an indicator for nonattainment status.
- **Second Stage**:
$$\Delta \text{HousingPrice}_c = \alpha_1 \Delta \widehat{\text{Pollution}}_c + \alpha_2 \Delta X_c + \mu_c$$
- **Choice of Instrument Timing**: The authors use nonattainment status from 1975–1976 as the instrument, because:
    - It is close to 1980, minimizing migration responses.
    - Treatment and control counties are more comparable.

Here are the empirical results. Read the table and think about answering the following questions.

TABLE 5
INSTRUMENTAL VARIABLES ESTIMATES OF THE EFFECT OF 1970–80 CHANGES IN TSPs POLLUTION ON CHANGES IN LOG HOUSING VALUES

	(1)	(2)	(3)	(4)
	A. TSPs Nonattainment in 1975 or 1976			
Mean TSPs (1/100)	−.362	−.213	−.266	−.202
	(.152)	(.096)	(.104)	(.090)
Sample size	988	983	983	983
	B. TSPs Nonattainment in 1975			
Mean TSPs (1/100)	−.350	−.204	−.228	−.129
	(.150)	(.099)	(.102)	(.084)
Sample size	975	968	968	968
	C. TSPs Nonattainment in 1970, 1971, or 1972			
Mean TSPs (1/100)	.072	−.032	−.050	−.073
	(.058)	(.042)	(.041)	(.035)
Sample size	988	983	983	983
County Data Book covariates	no	yes	yes	yes
Flexible form of county covariates	no	no	yes	yes
Region fixed effects	no	no	no	yes

NOTE.—See the notes to previous tables. The coefficients are estimated using 2SLS. The first row of panels A–C indicates which instrument is used. From panels A to C, the instruments are an indicator equal to one if the county was nonattainment for TSPs in either 1975 or 1976, an indicator equal to one if the county was nonattainment for TSPs in 1975, and an indicator that equals one if the county was nonattainment for TSPs in either 1970, 1971, or 1972, respectively. Standard errors (in parentheses) are estimated using the Eicker-White formula to correct for heteroskedasticity.

$$lnPrice = \alpha \frac{Pollution}{100}$$

$$\frac{dPrice}{Price} = \frac{\alpha}{100} dPollution$$

What is the difference between Panel A and Panel C?

- The choice of instrumental variable year differs.
  - **Panel A** uses air quality attainment status from **1975–1976** as the instrument.
  - **Panel C** uses air quality attainment status from **1970–1972** as the instrument.

Interpreting the regression results: Taking column (4) as an example. Which panel suggests a larger externality from air pollution?

- **Panel A** results: A one-unit increase in air pollution is associated with a **0.2% decrease** in housing prices.
- **Panel C** results: A one-unit increase in air pollution is associated with a **0.07% decrease** in housing prices.
- **Panel A** estimates a larger negative impact of pollution on housing prices (0.2% vs. 0.07%), indicating a greater perceived external cost of pollution.

The difference arises because the 1975–1976 attainment status is less contaminated by migration responses and provides a cleaner comparison between treatment and control counties, making Panel A the more reliable estimate.

Chapter 4 Public Goods

This chapter covers the Samuelson Rule for optimal provision, Lindahl prices as a market solution, and the Clarke mechanism for preference revelation. We analyze how to overcome free-rider problems in funding public goods.

4.1 Effective Supply: Samuelson's Rule

4.1.1 Definition of Public Goods

Definition 4.1.1: Public goods are characterized by:

- Non-excludability
- Non-rivalry in consumption

4.1: Goods Classification Based on Rivalry and Excludability

	Excludable	**Non-Excludable**
Rivalrous	Private Goods	Common Resources
	(e.g., food, clothing)	(e.g., forests, air)
Non-Rivalrous	Club Goods	Pure Public Goods
	(e.g., toll parks, libraries)	(e.g., national defense)

Classification of Roads as the specific example.

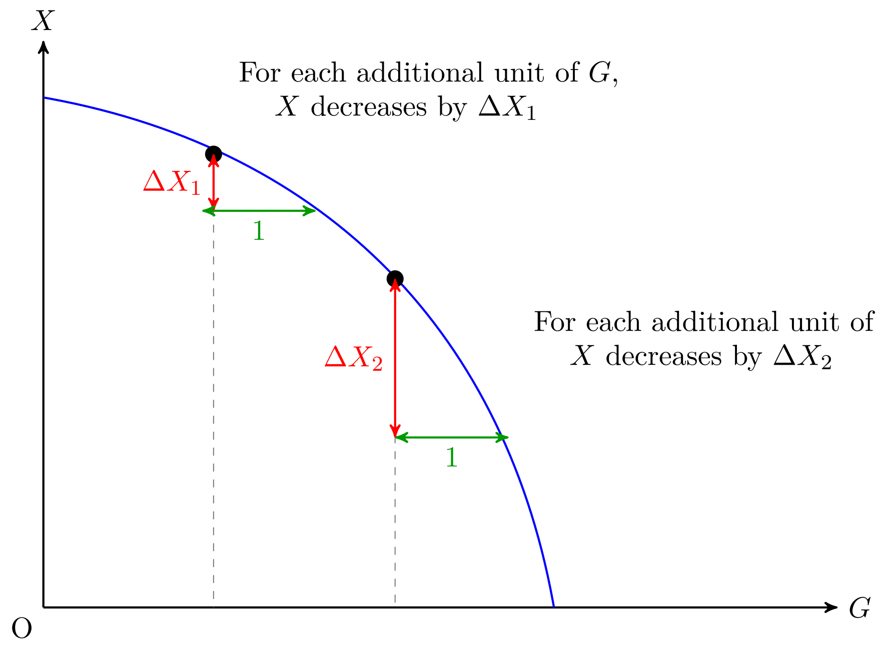
	Excludable	**Non-Excludable**
Rivalrous	Tolled Congested Road	Non-Tolled Congested Road
Non-Rivalrous	Tolled Uncongested Road	Non-Tolled Uncongested Road

4.1.2 First Best: The Samuelson Rule

Consider an economy with:

- Two representative consumers: A and B
- Two goods: X (private good) and G (public good)
- Consumer utility functions: $U_i(X, G), \quad i = A, B$
  - Special case (quasi-linear utility):
$$U_i(X, G) = X + u_i(G)$$
- Production possibility frontier: $F(X, G) = 0$
  - Special case (linear production):
$$aX + bG = \Gamma$$
  - Here, producing one unit of X requires a units of input, and producing one unit of G requires b units of input.
  - This can be rewritten as:
$$X + \frac{b}{a}G = \frac{\Gamma}{a}$$
  - Without loss of generality, we can simplify the production possibility frontier to:
$$X + cG = \Gamma$$
where $c = \frac{b}{a}$ represents the marginal rate of transformation between the private good and the public good.

Production Possibility Frontier



4.1: Production Possibility Frontier

For each additional unit of G produced, the output of X decreases by ΔX.

- When producing one more unit of G:

$$\Delta X \quad \text{(reduction in } X \text{ production)}$$

- What is the relationship between ΔX_1 and ΔX_2? Why?
  - Due to diminishing returns or increasing opportunity cost, typically $\Delta X_2 > \Delta X_1$. As more resources are allocated to producing G, the marginal cost of producing G increases, meaning more X must be sacrificed for each additional unit of G.

- The slope of the production possibility frontier is called the **Marginal Rate of Transformation (MRT)**:

$$MRT = \frac{MC_G}{MC_X}$$

  where MC_G is the marginal cost of producing one more unit of G, and MC_X is the marginal cost of producing one more unit of X.

- **Intuition:**
  - Producing one more unit of X requires a units of input.
  - Producing one more unit of G requires b units of input.
  - Therefore, producing one more unit of G means sacrificing $\frac{b}{a}$ units of X:

$$MRT = \frac{b}{a}$$

  This is the rate at which the economy can transform X into G given available resources and technology.

4.1.3 Optimal Provision of Private Goods: Assume Both X and G Are Private Goods

The Pareto optimal allocation satisfies:

$$\max \; \alpha_1 U_A(X_A, G_A) + \alpha_2 U_B(X_B, G_B)$$

$$\text{s.t.} \quad (X_A + X_B) + c(G_A + G_B) = \Gamma$$

The Lagrangian function is:

$$\mathcal{L} = \alpha_1 U_A(X_A, G_A) + \alpha_2 U_B(X_B, G_B) + \lambda \left[\Gamma - (X_A + X_B) - c(G_A + G_B)\right]$$

First-Order Conditions:

$$\begin{aligned} (X_A): &\quad \alpha_1 MU_{AX} = \lambda, &\qquad (G_A): &\quad \alpha_1 MU_{AG} = \lambda c \\ (X_B): &\quad \alpha_2 MU_{BX} = \lambda, &\qquad (G_B): &\quad \alpha_2 MU_{BG} = \lambda c \end{aligned}$$

Simplifying these conditions yields:

$$\frac{MU_{AG}}{MU_{AX}} = \frac{MU_{BG}}{MU_{BX}} = c$$

Interpretation:

- The optimal solution satisfies: **Marginal Rate of Substitution (MRS) equals Marginal Rate of Transformation (MRT)**:

$$MRS = MRT$$

- **Quasi-linear utility case:**

$$U_i(X, G) = X + u_i(G)$$

The first-order conditions simplify to:

$$u'_A(G_A) = u'_B(G_B) = c$$

- This implies: Marginal utility equals marginal cost for each consumer. In the quasi-linear setting, the optimal quantity of G for each individual is determined where their marginal benefit from G equals the marginal cost c.

Quasi-Linear Utility Case

Consider a private good G. Consumer A's demand curve is given by $D_A = u'_A(G_A)$, and consumer B's demand curve is $D_B = u'_B(G_B)$. The market supply curve is $S = MC(G)$, where $G = G_A + G_B$ is the total quantity. What are the equilibrium price and quantity?

Deriving the Demand Curve

Each consumer solves the utility maximization problem:

$$\max_{X,G} \ X + u(G) \quad \text{s.t.} \quad X + pG = I$$

Substituting the budget constraint into the objective function:

$$\max_{G} \ I - pG + u(G)$$

The first-order condition gives:

$$p = u'(G)$$

Thus, each consumer's demand curve is simply their marginal utility curve: $D_i = u'_i(G_i)$.

Deriving the Supply Curve

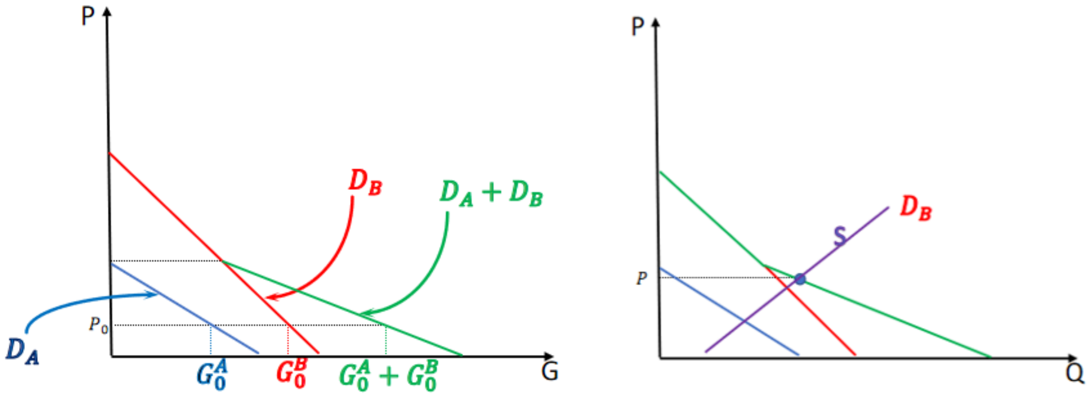
Firms maximize profit:

$$\max_{G} \ pG - C(G)$$

The first-order condition gives:

$$p = C'(G) = MC(G)$$

Thus, the market supply curve is the marginal cost curve.



4.2: Graph Solution for Private Good

e.g. 4: Intuitive Example
If rice costs 1 yuan per bowl, A demands 1 bowl, and B demands 2 bowls. How many bowls do they demand in total?

$$\text{Total quantity demanded} = 1 + 2 = 3 \text{ bowls}$$

This illustrates horizontal summation: at $P = 1$, $G_A = 1$, $G_B = 2$, so total market demand $G = 3$.

- Social Demand Curve for Private Goods:

  The social demand curve is obtained by **horizontal summation** of individual demand curves: for a given price P, we add the quantities demanded by each consumer.

$$G(P) = G_A(P) + G_B(P)$$

- At market equilibrium:

$$P = u'_A(G_A) = u'_B(G_B) = MC(G_A + G_B)$$

4.1.4 Optimal Provision of Public Goods: Assume G is a Public Good

Consider an economy with two consumers (A and B) and two goods: a private good X and a public good G. The public good is non-rival and non-excludable, meaning both consumers enjoy the same quantity G.

The Pareto optimal allocation solves:

$$\max \ \alpha_1 U_A(X_A, G) + \alpha_2 U_B(X_B, G)$$

$$\text{s.t.} \quad (X_A + X_B) + cG = \Gamma$$

where c is the marginal cost of producing one unit of the public good in terms of the private good (i.e., the marginal rate of transformation $MRT = c$).

The Lagrangian function is:

$$\mathcal{L} = \alpha_1 U_A(X_A, G) + \alpha_2 U_B(X_B, G) + \lambda \left[\Gamma - (X_A + X_B) - cG\right]$$

First-Order Conditions

$$\begin{aligned} (X_A): &\quad \alpha_1 MU_{AX} = \lambda \\ (X_B): &\quad \alpha_2 MU_{BX} = \lambda \\ (G): &\quad \alpha_1 MU_{AG} + \alpha_2 MU_{BG} = \lambda c \end{aligned}$$

Dividing the condition for G by λ and using the conditions for X_A and X_B to substitute for λ:

$$\frac{\alpha_1 MU_{AG}}{\alpha_1 MU_{AX}} + \frac{\alpha_2 MU_{BG}}{\alpha_2 MU_{BX}} = c$$

This simplifies to:

$$\frac{MU_{AG}}{MU_{AX}} + \frac{MU_{BG}}{MU_{BX}} = c$$

Interpretation

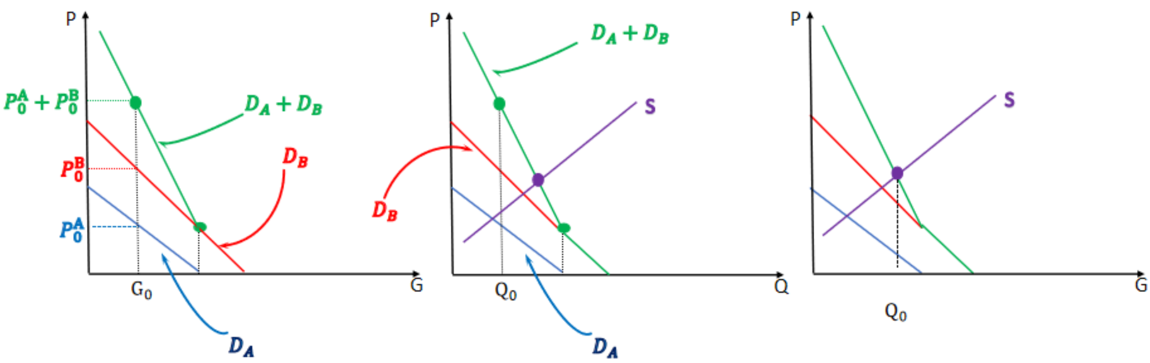
- The optimal solution satisfies: **The sum of marginal rates of substitution equals the marginal rate of transformation**:

$$MRS_A + MRS_B = MRT$$

- This is known as the The **Samuelson Rule** (Samuelson 1954, *Review of Economics and Statistics*).
- Intuition: For a public good, the social marginal benefit is the sum of all individuals' marginal benefits. Optimal provision requires that this sum equals the marginal cost of production.

Quasi-Linear Utility Case

Consider a public good G. Consumer A's demand curve (marginal benefit) is given by $D_A = u'_A(G)$, and consumer B's demand curve is $D_B = u'_B(G)$. The market supply curve is $S = MC(G)$, where G is the total quantity of the public good provided (the same quantity for both consumers).



4.3: Graph Solution for Public Good

e.g. 5: Intuitive Example
In a classroom:

- **A is willing to pay 1 yuan for 1 hour of air conditioning**
- **B is willing to pay 2 yuan for 1 hour of air conditioning**

Question: How much are they jointly willing to pay for 1 hour of air conditioning?

$$\text{Total willingness to pay} = 1 + 2 = 3 \text{ yuan}$$

In the classroom example:

- At $G = 1$ hour, $u'_A(1) = 1$ yuan, $u'_B(1) = 2$ yuan
- Total marginal benefit $= 1 + 2 = 3$ yuan
- If the marginal cost of providing 1 hour of air conditioning is 3 yuan, then $G^* = 1$ hour is optimal

- Social Demand Curve for Public Goods

  The social demand curve is obtained by **vertical summation** of individual demand curves:

$$\text{At a given quantity } G, \quad P_{\text{total}}(G) = P_A(G) + P_B(G)$$

- Optimal Provision Condition:

  At the socially optimal quantity G^*:

$$u'_A(G^*) + u'_B(G^*) = MC(G^*)$$

4.1.5 Comments on the Samuelson Rule

The government is assumed to have perfect information about consumer preferences.

It does not consider that providing public goods may require taxation, which can introduce market distortions.

It focuses only on the optimal quantity of the public good, not on who provides it. Therefore, the derivation uses only non-rivalry, not non-excludability.

When considering who should pay for the public good, non-excludability becomes relevant, leading to the "free-rider problem."

4.2 A Hypothetical Market Solution: Lindahl Prices

4.2.1 What is Lindahl Prices?

Objective: How can we achieve the optimal provision of public goods through a market mechanism?

The marginal utility of the public good differs across consumers. Let each person pay a different price that reflects their willingness to pay (marginal utility). Under Lindahl prices, each consumer demands the same quantity of the public good.

Such a set of personalized prices is called "**Lindahl Prices**." At equilibrium, is the quantity of the public good necessarily optimal?

e.g. 6: Example of Lindahl Prices

Consider two neighbors, A and B, who share a staircase and need to decide on paying for the electricity for a communal light.

- The electricity cost per hour is fixed and given.
- A and B must decide:
  - The share of the cost each will pay. Let h be the share paid by A, and $1 - h$ the share paid by B.
  - The number of hours the light will be on each day, denoted Q.

Key Questions

1. Does there exist a cost share h^* such that A and B demand the same quantity of light Q?
2. If such an h^* exists, is the resulting quantity Q^* socially optimal? That is, does it satisfy the Samuelson Rule?

4.2.2 General Case of Lindahl Prices

Consider an economy with:

- **Two representative consumers:** A and B
- **Two goods:**
  - X: Private good
  - G: Public good (non-rival and non-excludable)
- **Consumers:**
  - Utility functions: $U_i(X, G)$ for $i = A, B$
  - Special case (quasi-linear utility):

$$U_i(X, G) = X + u_i(G)$$

  - Initial endowments (interpreted as income): I_A and I_B

- **Production:**
  - Assume the marginal cost of producing X is 1
  - Assume the marginal cost of producing G is c
  - In a perfectly competitive market, the price of X is 1, and the price of G is c

Under the Lindahl pricing scheme, each consumer pays a personalized price for the public good G:

- Consumer A pays τ_A per unit of G
- Consumer B pays τ_B per unit of G
- The personalized prices must cover the marginal cost of production:

$$\tau_A + \tau_B = c$$

Key Questions

1. Can we find personalized prices τ_A^* and τ_B^* such that both consumers demand the same quantity G^* of the public good?
2. If such prices exist, is the resulting quantity G^* socially optimal? That is, does it satisfy the Samuelson Rule?

4.2.3 Consumer Optimization

Each consumer chooses their consumption of the private good X_i and the public good G (which is consumed jointly) subject to their budget constraint.

For consumer A:

$$\max_{X_A,G} U_A(X_A, G) \quad \text{s.t.} \quad X_A + \tau_A G = I_A$$

The Lagrangian function is:

$$L = U_A(X_A, G_A) + \lambda_A(I_A - X_A - \tau_A G_A)$$

First-Order Conditions for A

$$\begin{aligned} (X_A): &\quad MU_{AX}(X_A, G_A) = \lambda_A \\ (G_A): &\quad MU_{AG}(X_A, G_A) = \lambda_A \tau_A \end{aligned}$$

Dividing the two conditions, we obtain:

$$\boxed{\frac{MU_{AG}(X_A, G_A)}{MU_{AX}(X_A, G_A)} = \tau_A}$$

Similarly, for consumer B:

$$\boxed{\frac{MU_{BG}(X_B, G_B)}{MU_{BX}(X_B, G_B)} = \tau_B}$$

In a Lindahl equilibrium, two key conditions must hold:

- Both consumers demand the same quantity of the public good:

$$G_A = G_B = G^*$$

- The personalized prices cover the marginal cost of production:

$$\tau_A + \tau_B = c$$

Substituting the equilibrium conditions into the first-order conditions:

$$\frac{MU_{AG}(X_A, G^*)}{MU_{AX}(X_A, G^*)} + \frac{MU_{BG}(X_B, G^*)}{MU_{BX}(X_B, G^*)} = \tau_A + \tau_B = c$$

Therefore:

$$\boxed{\frac{MU_{AG}}{MU_{AX}} + \frac{MU_{BG}}{MU_{BX}} = c}$$

Interpretation

- This is exactly the **Samuelson Rule** for the optimal provision of a public good:

$$MRS_A + MRS_B = MRT$$

- The Lindahl equilibrium achieves the socially optimal (Pareto efficient) allocation.
- The personalized prices τ_A and τ_B reflect each consumer's marginal rate of substitution at the optimum.

Quasi-Linear Utility Case

Under quasi-linear utility $U_i(X, G) = X + u_i(G)$, the Lindahl prices simplify to:

$$\tau_A = u'_A(G^*), \quad \tau_B = u'_B(G^*)$$

The Samuelson Rule becomes:

$$u'_A(G^*) + u'_B(G^*) = c$$

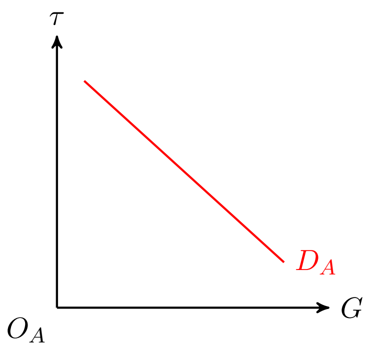
e.g. 7: Connect to the Staircase Light Example

- Let G be the number of hours the stairwell light is on.
- The marginal utility (benefit) of the light to A and B is $u'_A(G)$ and $u'_B(G)$, respectively.
- The electricity cost per hour is c yuan.
- The socially optimal quantity G^* satisfies the condition that social marginal benefit equals social marginal cost:

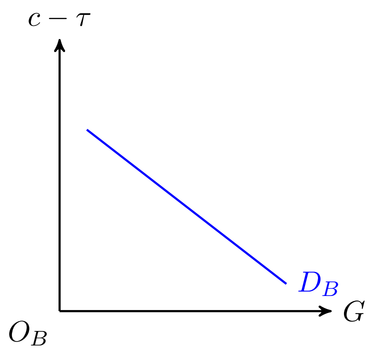
$$u'_A(G^*) + u'_B(G^*) = c$$

- A hypothetical market (Lindahl pricing) can achieve this social optimum:
  - A pays $u'_A(G^*)$ per hour
  - B pays $u'_B(G^*)$ per hour
  - Both A and B demand exactly G^* hours of light

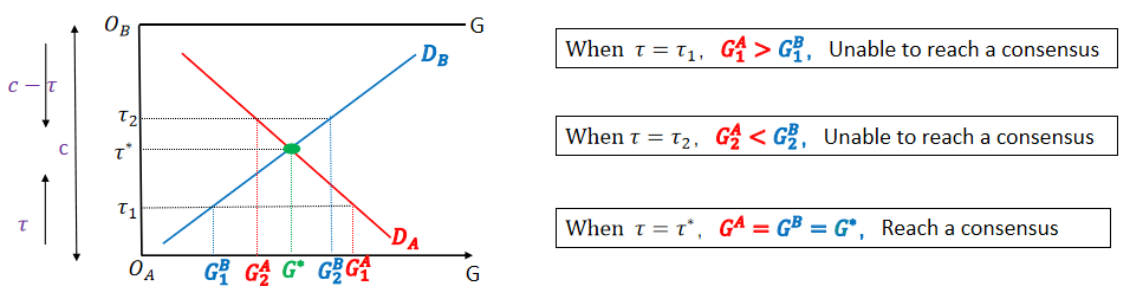
Thus, the Lindahl mechanism provides a way to decentralize the efficient provision of a public good, even though no real market for it exists.
Next, we use graphics to demonstrate the Lindahl pricing mechanism.



4.4: Demand Curve of A



4.5: Demand Curve of B



4.6: The process of forming a consensus

4.2.4 Comments on Lindahl Prices

Theoretical Achievement:

- In theory, the Lindahl pricing mechanism can achieve the socially optimal allocation.
- The derivation only relies on the non-rivalry property of public goods. It does not require non-excludability.

Practical Difficulties:

- In reality, Lindahl prices are difficult to implement due to non-excludability.
- The free-rider problem emerges: individuals have no incentive to reveal their true preferences.
- The mechanism requires knowing each consumer's marginal utility:

$$\tau_i = MU_i(G)$$

  But consumers will understate their true marginal utility to avoid paying.

How to Elicit Truthful Preferences?

- Edward Clarke proposed a mechanism design that incentivizes individuals to reveal their true preferences for public goods.
- This is known as the **Clarke (Groves) Mechanism** or the **pivotal mechanism**.
- It creates a situation where truth-telling is a dominant strategy for each individual.

We now turn to this mechanism design approach.

4.3 The Clarke (Groves) Mechanism Design

4.3.1 Model Setup

- There are n consumers. For simplicity, consider $n = 3$.
- Consider the provision of a public good.
  - General case: The quantity of the public good G is continuous.
  - Special case :(in our setting) G is binary, taking only two values:

$$G \in \{0, 1\}$$

E.g.: turning on the air conditioning or not, installing a light or not.

- The public good has a total cost c.
- Consumers share the cost. Let c_1, c_2, c_3 be the payments by consumers 1, 2, and 3, satisfying:

$$c_1 + c_2 + c_3 = c$$

- The public good provides utility u_1, u_2, u_3 to consumers 1, 2, and 3, respectively.

- Social efficiency condition: The public good should be provided if and only if:

$$u_1 + u_2 + u_3 > c$$

The Revelation Problem:

- Consumers know their own utility u_i from the public good, but they have no incentive to reveal it truthfully.
- If asked, a consumer may understate their utility to avoid paying, or overstate it to get the good provided at others' expense.
- Key question: How can we design a mechanism that induces consumers to reveal their true utilities?

This is the central challenge of mechanism design for public goods.

4.3.2 Description of the Mechanism

Step 1: Setting Cost Shares

- The government first sets cost shares c_i for each consumer, satisfying:

$$\sum_{i=1}^{n} c_i = c$$

  where c is the total cost of providing the public good.

Step 2: Eliciting Preferences

- Each consumer is asked to report their *net utility* from the public good, denoted s_i.
- The true net utility for consumer i is:

$$r_i = u_i - c_i$$

  where u_i is the true gross utility.
- Consumers may not report truthfully unless given proper incentives.

Step 3: Decision Rule and Clarke Tax

- The public good is provided if and only if the sum of reported net utilities is positive:

$$\sum_{i=1}^{n} s_i > 0 \quad \Rightarrow \quad \text{Provide}$$

$$\sum_{i=1}^{n} s_i \leq 0 \quad \Rightarrow \quad \text{Do not provide}$$

- A consumer is called a **pivotal person** (or pivotal agent) if their report changes the social decision

- Pivotal consumers must pay a tax, known as the **Clarke tax**.

Definition 4.3.1: Pivotal Person/ Agent

An individual is called the "pivotal agent" if his/her reported preference changes the social decision (whether to provide the public good or not). This consumer j is pivotal if the decision with and without his/her report differs.

Scenario	Without j	With j	Tax for j
1	$\sum_{i \neq j} s_i > 0$ (should provide)	$\sum_i s_i < 0$ (should not provide)	$\sum_{i \neq j} s_i$ (net loss of others)
2	$\sum_{i \neq j} s_i < 0$ (should not provide)	$\sum_i s_i > 0$ (should provide)	$-\sum_{i \neq j} s_i$ (net loss of others)

4.2: Clarke Tax for a Pivotal Consumer j

Important Note:

- The tax collected from the pivotal consumer is not distributed among the other $n - 1$ consumers.
- Instead, it is paid to someone outside the group (e.g., the government) and does not affect the utility of the n consumers.
- This ensures that the tax does not create additional incentives for strategic behavior among the consumers.

4.3.3 Individual Optimal Choice

Key Result: Reporting one's true utility is a dominant strategy. Regardless of what others report, revealing true preferences is (one of) the optimal choices for each individual.
Consider consumer j's decision problem:

- By reporting different net utilities s_j, consumer j can influence whether the public good is provided ($G = 1$) or not ($G = 0$).
- Given the sum of others' reports $\sum_{i \neq j} s_i$, consumer j's utility can be summarized as:

	$\sum_{i \neq j} s_i > 0$	$\sum_{i \neq j} s_i < 0$
$G = 1$	r_j	$r_j + \sum_{i \neq j} s_i$
$G = 0$	$-\sum_{i \neq j} s_i$	0

4.3: Consumer j's Utility Under Different Scenarios

Given $\sum_{i \neq j} s_i$, consumer j's utility can be expressed as:

$$U_j = \left(r_j + \sum_{i \neq j} s_i\right) \cdot G - \sum_{i \neq j} s_i \cdot \mathbf{I}\left(\sum_{i \neq j} s_i > 0\right)$$

where $\mathbf{I}(\cdot)$ is the indicator function:

$$\mathbf{I}\left(\sum_{i \neq j} s_i > 0\right) = \begin{cases} 1, & \text{if } \sum_{i \neq j} s_i > 0 \\ 0, & \text{if } \sum_{i \neq j} s_i < 0 \end{cases}$$

Interpretation:

- The first term $\left(r_j + \sum_{i \neq j} s_i\right) \cdot G$ captures the total social surplus if the public good is provided.
- The second term $-\sum_{i \neq j} s_i \cdot \mathbf{I}(\sum_{i \neq j} s_i > 0)$ is the Clarke tax paid when j is pivotal.
- This formulation makes it clear that consumer j's optimal strategy is to report $s_j = r_j$, because any misreport would either:
  - Not change the outcome but risk becoming pivotal and paying a tax, or
  - Change the outcome in a way that reduces their own utility.

Given $\sum_{i \neq j} s_i$, consumer j's utility can be expressed as:

$$U_j = \left(r_j + \sum_{i \neq j} s_i\right) \cdot G - \sum_{i \neq j} s_i \cdot \mathbf{1}\left(\sum_{i \neq j} s_i > 0\right)$$

where $\mathbf{1}(\cdot)$ is the indicator function.

Case 1: If $r_j + \sum_{i \neq j} s_i > 0$, then setting $G = 1$ maximizes utility.

How to make $G = 1$?
The public good is provided $(G = 1)$ if and only if:

$$s_j + \sum_{i \neq j} s_i > 0$$

In this case, reporting truthfully $s_j = r_j$ ensures that this condition holds.

Case 2: If $r_j + \sum_{i \neq j} s_i < 0$, then setting $G = 0$ maximizes utility.

How to make $G = 0$?
The public good is not provided $(G = 0)$ if and only if:

$$s_j + \sum_{i \neq j} s_i < 0$$

In this case, reporting truthfully $s_j = r_j$ ensures that this condition holds.
Regardless of others' choices, reporting one's true utility maximizes individual utility!

4.3.4 Comments on the Clarke Mechanism

- The Clarke mechanism can achieve efficient provision of public goods.
  - The decision rule $\sum_i s_i > 0$ is equivalent to:
  $$\sum_i u_i > \sum_i c_i = c$$
  - This is exactly the condition that social marginal benefit exceeds social marginal cost.
- However, the presence of the Clarke tax may lead to inefficiencies.
  - The tax itself is a transfer that does not affect the provision decision directly, but it may distort participation incentives.
- The cost shares c_i must be determined in advance.
  - These predetermined shares may not align with the actual utility consumers derive from the public good.
  - As a result, some consumers may end up with negative net utility ($r_i = u_i - c_i < 0$).
  - The provision of the public good could make some individuals worse off.
- Individuals may not be willing to participate in the mechanism.
  - If consumers anticipate that they might become pivotal and have to pay a tax, or that the cost shares are unfair, they may choose not to participate.
  - Voluntary participation is not guaranteed, which undermines the mechanism's feasibility.

In summary, while the Clarke mechanism solves the preference revelation problem in theory, it faces practical challenges related to fairness, participation, and potential inefficiencies from the tax itself.

Chapter 5 Social Insurance

This chapter covers health insurance with information asymmetry and pension insurance under OLG framework. Finally, we review three papers on JPubE.

5.1 Health Insurance

In this section, we will analyze the economics of health insurance markets.

- Theoretical Foundation 1: Expected Utility Theory
  - Individuals make decisions under uncertainty.
  - They maximize expected utility rather than certain outcomes.
  - This provides the basis for understanding why people buy insurance and how they value risk reduction.
- Theoretical Foundation 2: Information Asymmetry
  - In insurance markets, buyers and sellers have different information.
  - This leads to two classic market failures:
    * Adverse Selection: Individuals with higher health risks are more likely to buy insurance, driving up premiums and potentially causing the market to unravel.
    * Moral Hazard: Once insured, individuals may engage in riskier behavior or consume more healthcare than they would without insurance.

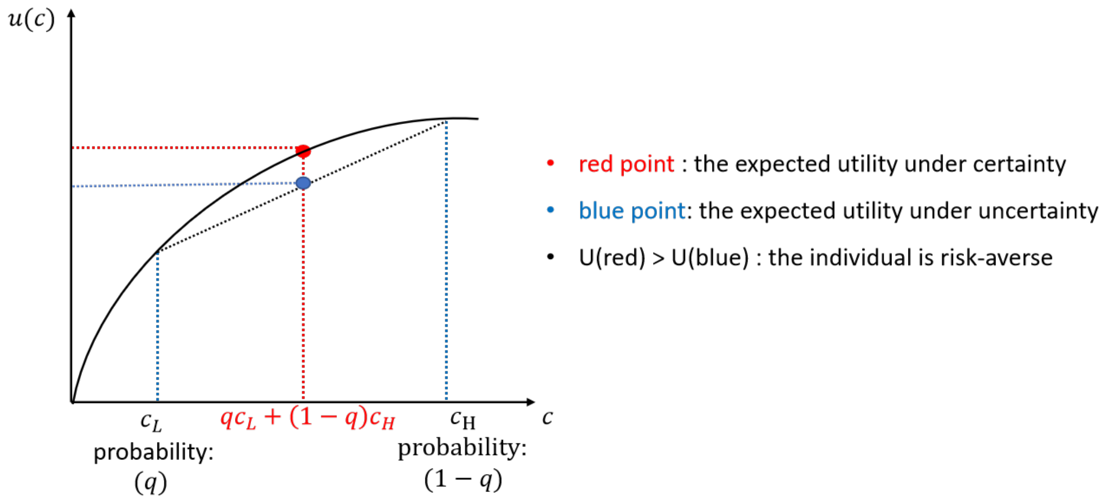
5.1.1 Expected Utility Theory

Consider an individual with a utility function $u(c)$, where c represents consumption or wealth.

- With probability q, consumption is low: $c = c_L$
- With probability $1 - q$, consumption is high: $c = c_H$

Expected Utility:

$$EU = q \cdot u(c_L) + (1 - q) \cdot u(c_H)$$

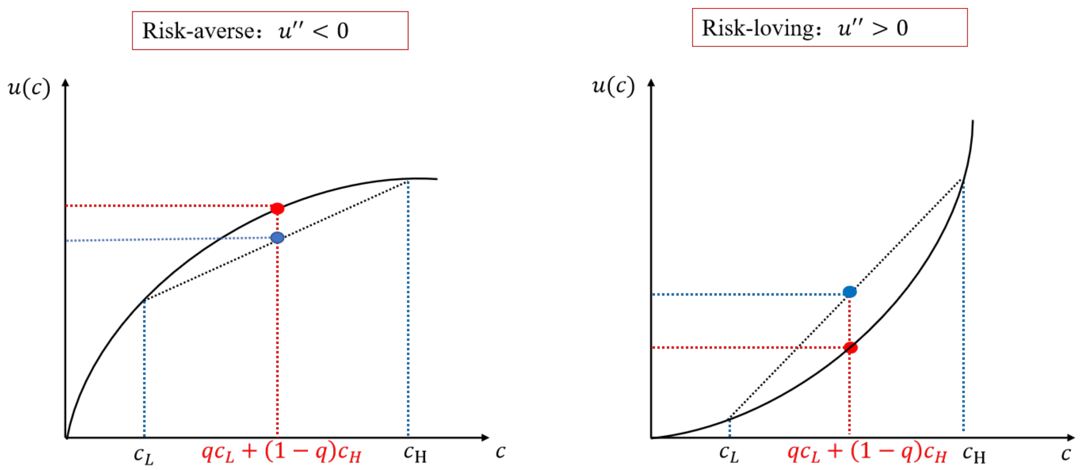


5.1: Graphic Display of Expected Utility

Question for Reflection:
How is an individual's attitude toward risk reflected in the expected utility framework?

Hints:

- Risk attitude is captured by the shape of the utility function $u(c)$.
  - Risk-averse: $u''(c) < 0$ (concave function)
  - Risk-neutral: $u''(c) = 0$ (linear function)
  - Risk-loving: $u''(c) > 0$ (convex function)
- The curvature of $u(c)$ determines how much the individual dislikes uncertainty.
- For a given lottery, a risk-averse individual would prefer a certain outcome equal to the expected value of the lottery over the lottery itself.



5.2: Risk Preference

When the utility function is concave ($u'' < 0$), the individual is risk-averse, meaning the utility of expected wealth exceeds the expected utility of wealth:

$$qu(c_L) + (1-q)u(c_H) < u(qc_L + (1-q)c_H)$$

Intuitively, diminishing marginal utility means that the loss in utility from a bad outcome (c_L) is larger than the gain in utility from an equally likely good outcome (c_H), making uncertainty undesirable

5.1.2 A Simple Model of Health Insurance

5.1.2.1 Model Setup

Representative Household:

- Income: W
- Probability of illness: q. If ill, medical expenses are d.
- Utility function: $u(c)$, where c is consumption net of medical expenses.

Insurance:

- The household can purchase b units of insurance coverage.
- If ill, the insurance reimburses b of the medical expenses.
- The insurance premium is p (paid regardless of health state).

Fair Premium (Actuarially Fair Insurance):

- A premium is called "fair" if it yields zero expected profit for the insurance company.
- The insurance company's expected profit is:

$$(1-q) \cdot p + q \cdot (p-b) = 0$$

- Solving for p:

$$p - qb = 0 \quad \Rightarrow \quad p = qb$$

- The fair premium equals the expected payout.

5.1.2.2 How Much Insurance Will the Household Purchase?

If the household can choose b, the optimal b can maximize expected utility:

$$\max_b \; EU = q \cdot u(W - qb - d + b) + (1-q) \cdot u(W - qb)$$

First-order condition with respect to b:

$$q \cdot u'(W - qb - d + b) \cdot (1-q) + (1-q) \cdot u'(W - qb) \cdot (-q) = 0$$

Simplifying:

$$u'(W - qb - d + b) = u'(W - qb)$$

Since u' is strictly decreasing (risk aversion), this implies:

$$W - qb - d + b = W - qb$$

Solving yields:

$$\boxed{b = d}$$

Conclusion: Under fair premium ($p = qb$), the household's optimal choice is to purchase full insurance ($b = d$). The household is perfectly insured, eliminating all consumption risk.

5.1.2.3 Introducing Heterogeneity

Now assume there are two types of individuals:

- Low-risk type: probability of illness q_L
- High-risk type: probability of illness q_H
- Assume $q_L < q_H$

Under perfect information, the insurance company can distinguish between low-risk and high-risk individuals.

- Low-risk individuals pay fair premium: $p_L = dq_L$
- High-risk individuals pay fair premium: $p_H = dq_H$
- Both types purchase full insurance ($b = d$).

5.1.2.4 Introducing Information Asymmetry

Now consider information asymmetry:

- The insurance company **cannot** distinguish between low-risk and high-risk individuals.
- If the company continues to offer two full-insurance contracts (one with premium dq_L and one with premium dq_H), then:
  - Everyone will choose the cheaper contract with premium dq_L.
  - This contract becomes unsustainable because it attracts both types but is priced for low risks only.
- Under asymmetric information, two types of equilibrium may arise:
  - Pooling equilibrium: Both types buy the same insurance contract.
  - Separating equilibrium: Different types self-select into different contracts designed to reveal their risk type.

5.1.3 Equilibrium Under Asymmetric Information

Under asymmetric information, two types of equilibrium may arise in insurance markets:

1. **Pooling Equilibrium**

   - A single insurance contract is offered, and both high-risk and low-risk individuals purchase the same contract.
   - The premium is set based on the average risk in the population:

     $$p = d \cdot (\lambda q_H + (1 - \lambda) q_L)$$

     where λ is the proportion of high-risk individuals.
   - Problem: Low-risk individuals are overcharged (pay more than their fair premium), while high-risk individuals are subsidized.
   - Result: It can be shown that **no pooling equilibrium exists in a perfectly competitive insurance market**. A competitor can always offer a contract that attracts only low-risk individuals, making positive profits and breaking the pooling contract.

2. **Separating Equilibrium**

   - Two different contracts are offered, designed so that different risk types self-select into different contracts:
     - Contract H: High premium, but offers *full insurance* ($b = d$). This contract is attractive to high-risk individuals.
     - Contract L: Low premium, but offers *partial insurance* ($b < d$). This contract is designed for low-risk individuals.
   - High-risk individuals choose Contract H because they value full insurance more due to their higher risk.
   - Low-risk individuals choose Contract L because they prefer a lower premium over full coverage.
   - Key observation: In a separating equilibrium, **low-risk individuals are not fully insured**. This represents an efficiency loss compared to the full-information case.
   - Existence: In a perfectly competitive market, a separating equilibrium *may* exist under certain conditions, provided that the contracts are incentive-compatible and no profitable deviation exists.

5.1.4 Moral Hazard

After purchasing insurance, individuals may change their behavior:

- Reduced precaution against entering the adverse state (e.g., less health prevention).
- Increased odds of staying in the adverse state (e.g., unemployment insurance).
- Increased expenditures when in the adverse state (e.g., health insurance).

Of course, social insurance is not completely without shortcomings and also requires trade-offs. Its advantages and disadvantages can be summarized as follows

- Benefit: Smoothing consumption across different states (healthy vs. sick, employed vs. unemployed).
- Cost: Moral hazard — individuals may take less effort to avoid the adverse state or consume more once insured.

How to Mitigate Moral Hazard?

- Partial insurance: Not covering 100% of losses (e.g., co-payments, deductibles) to maintain incentives for precaution.
- Strengthened monitoring: Better oversight and regulation to reduce opportunistic behavior.

5.1.5 Rationale for Social Insurance: The Case of Health Insurance

- Mitigating adverse selection: Mandatory participation eliminates the adverse selection spiral.
  - Redistribution function: Same premium for all; low-risk individuals implicitly subsidize high-risk individuals.
  - From a government perspective, health risks are largely exogenous, so redistributing toward high-risk individuals may be socially desirable.
- Lower administrative costs: Social insurance systems typically have lower overhead than private insurance.
- Basic healthcare as a right: Access to essential medical services is considered a fundamental entitlement.
- Redistribution: Social insurance inherently redistributes income from the healthy to the sick, and from the rich to the poor.
- Externalities: Health care has public good aspects — e.g., prevention and treatment of infectious diseases benefit society as a whole.
- Myopia: Individuals may be short-sighted and underinsure against future risks without mandatory coverage.

5.2 Pension Insurance

Imagine you are 70 years old and can no longer work. How would you support yourself? Would you have saved enough? Would your family take care of you? These questions lie at the heart of the economics of aging and retirement.

An individual's ability to work naturally declines with age, yet the need to consume does not disappear after retirement. The standard life cycle model, developed by *Franco Modigliani*, who received the Nobel Prize in Economics in 1985 for this contribution, predicts that

- In the absence of any government program, rational individuals will save during their working years to finance consumption during retirement.
- This is the classic "consumption smoothing" behavior: individuals save when income is high and dissave when income is low.

In practice, however, without social pension systems, many people continue working until they are physically unable to do so, often until death, and then rely on family members for support. As a result, elderly poverty rates tend to be high in the absence of formal retirement programs. This gap between the theoretical prediction of forward-looking saving behavior and the reality of insufficient private provision motivates the existence of social security and pension systems. We now turn to the economics of pensions, exploring why governments intervene and how different pension systems work.

5.2.1 Rationale for Pension Systems

There are several economic justifications for the existence of public pension systems:
Individual Failures:

- Individuals may be *myopic* (short-sighted) and fail to save adequately for retirement, leading to insufficient private provision.

Market Failures:

- Adverse selection in private annuity markets: Insurance companies cannot distinguish between individuals with low and high life expectancy, leading to market inefficiencies or even market collapse.

Redistribution:

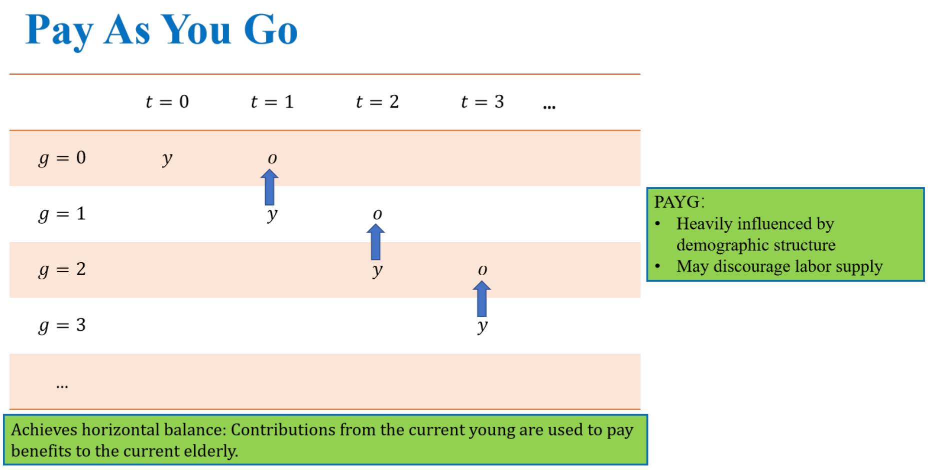
- Pension systems can redistribute income *within* a generation (intra-generational redistribution) and *across* generations (inter-generational redistribution).

5.2.2 Two Main Models of Pension Fund Management

5.2.2.1 Definition and Key Features

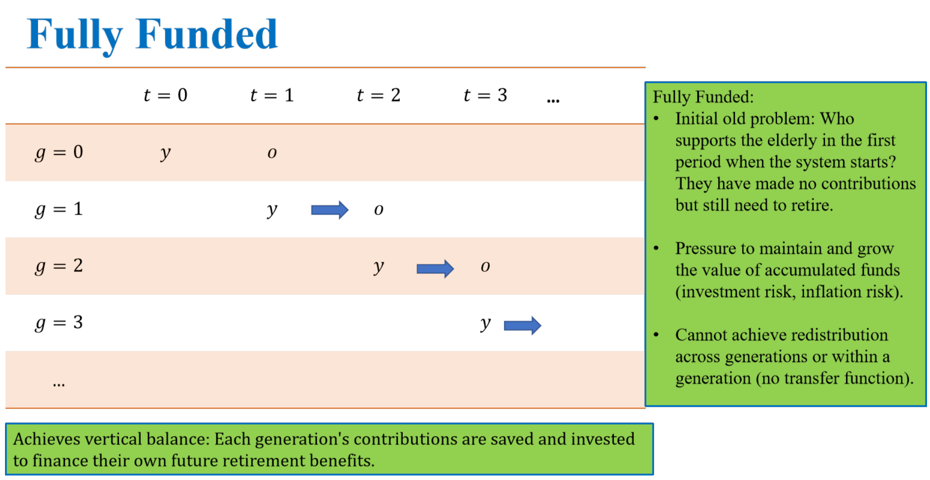
Pension systems can be organized in two fundamentally different ways:

- Pay-as-you-go (PAYG) System:
  - The benefits paid to current retirees come directly from the contributions of current workers.
  - There is an *intergenerational link*: each generation's pensions are financed by the next generation.
  - Current benefits = Current contributions.



5.3: PAYG System

- Fully Funded System:
  - Workers' contributions are invested in financial assets, and these assets (plus returns) are used to pay for their own future retirement benefits.
  - There is *no intergenerational link*: each generation finances its own pensions.
  - Current benefits = Past contributions + Investment returns on past contributions.



5.4: Fully Founded System

5.2.2.2 Model Setup

Consider a two-period **Overlapping Generations (OLG) model**. Generation t is born in period t and becomes old in period $t+1$. Each member of generation t earns a wage w_t, and the population size of generation t is N_t.

Pay-as-you-go (PAYG) System

- The first generation of retirees receives benefits without having contributed (initial old get a free transfer).

- For an individual born in period t:

$$T_t = \tau w_t \quad \text{(contribution when young)}$$
$$B_{t+1} = \frac{\tau w_{t+1} N_{t+1}}{N_t} \quad \text{(benefit when old)}$$

- This can be rewritten as:

$$B_{t+1} = \tau w_t \times \frac{w_{t+1} N_{t+1}}{w_t N_t} = \tau w_t (1+g)(1+n)$$

  where:
  - $g = \frac{w_{t+1} - w_t}{w_t}$ is the growth rate of wages (productivity growth)
  - $n = \frac{N_{t+1} - N_t}{N_t}$ is the population growth rate

- The implicit rate of return in a PAYG system is:

$$\varphi = (1+g)(1+n) - 1 = g + n + gn \approx n + g$$

Fully Funded System

- Each generation saves and invests, earning the market interest rate r.
- For an individual born in period t:

$$T_t = \tau w_t \quad \text{(contribution when young)}$$
$$B_{t+1} = T_t(1+r) = \tau w_t (1+r)$$

- The rate of return in a fully funded system is simply the market interest rate r.
- There is no intergenerational transfer: each generation finances its own retirement.

Comparison:

- PAYG return depends on wage growth g and population growth n: $\varphi \approx n + g$
- Funded return depends on the market interest rate r
- If $r > n + g$, the funded system offers a higher return; if $r < n + g$, PAYG is more attractive.
- This comparison is at the heart of debates over pension reform, especially in aging societies where n is low or negative.

5.2.3 Further Discussion on Pay-as-you-go Systems

- Samuelson (JPE 1958): In an overlapping generations economy with no capital and no ability to store goods (a "chocolate economy"), a pay-as-you-go pension system can generate a Pareto improvement because it enables intergenerational trade. *[This has the same effect as fiat money.]*

- Diamond (AER 1965): In an overlapping generations economy with capital and savings, a pay-as-you-go pension system generates a Pareto improvement *if and only if* $\varphi > r$, where φ is the implicit return of the PAYG system and r is the market interest rate.
  - If $\varphi < r$, the PAYG system redistributes income from all future generations to the initial retirees.

If $\varphi < r$: Intergenerational Redistribution

- **Gain for the initial old generation:**
  - Per capita benefit: $B_1 = \dfrac{\tau w_1 N_1}{N_0}$
  - Total gain: $N_0 B_1 = \tau w_1 N_1$
- **Loss for generation t ($t \geq 1$):**

$$V_t = T_t - \frac{B_{t+1}}{1+r} = \tau w_t - \frac{\tau w_t(1+\varphi)}{1+r}$$

- **Total loss across all future generations:**

$$\sum_{t=1}^{\infty} \frac{N_t V_t}{(1+r)^{t-1}} = \tau \sum_{t=1}^{\infty} \frac{N_1(1+n)^{t-1} w_1 (1+g)^{t-1}(r-\varphi)}{(1+r)^{t-1}} = \tau w_1 N_1$$

- This shows that the total loss of future generations exactly equals the initial gain of the first generation.

5.2.4 Pensions and Myopia

- Individuals live for two periods:
  - Period 1: Work, earn wage w, choose consumption c_1 and savings s
  - Period 2: Retire, consume savings with interest R: $c_2 = sR$
- Two types of individuals:
  - Rational: Utility $u(c_1) + \delta u(c_2)$
  - Myopic: Utility $u(c_1)$ (completely myopic)
  - Partially myopic: Utility $u(c_1) + \delta \beta u(c_2)$ with $0 < \beta < 1$
- From a social perspective, the true utility function is $u(c_1) + \delta u(c_2)$.

If you build a Three-Period Myopia Model, then

$$U_1 = u(c_1) + \delta\beta[u(c_2) + \delta u(c_3)]$$

$$U_2 = u(c_2) + \delta\beta u(c_3)$$

5.2.4.1 Savings Decision: Rational vs. Myopic

- Rational Individual (and Social Optimum):

$$\max_s \; u(w-s) + \delta u(sR)$$

First-order condition:

$$u'(w-s) = R\delta u'(sR)$$

This yields optimal savings $s^* > 0$.

e.g. 8

Example: If $\delta = 1$ and $R = 1$, then $s^* = \frac{w}{2}$.

- Myopic Individual:

$$\max_s \; u(w-s) \quad \text{s.t.} \quad c_2 = sR$$

Assuming a borrowing constraint (cannot borrow against future income), the optimal savings is:

$$s = 0$$

Implications:

- Myopic individuals save too little (or nothing) for retirement, leading to inadequate consumption in old age.
- This provides a rationale for mandatory pension systems that force individuals to save.
- The degree of myopia (β) determines how much intervention is needed.

5.2.4.2 Mandatory Pension Participation

If the government mandates participation in a pension system:

- Contribution in period 1: s^* (set equal to the socially optimal savings level)
- Pension benefit in period 2: $b = s^*R$

After joining the mandatory pension system:

- A **rational individual** chooses private savings s to maximize:

$$\begin{aligned} \max_s \quad & u(c_1) + \delta u(c_2) \\ \text{s.t.} \quad & c_1 + s + s^* = w \\ & c_2 = sR + s^*R \end{aligned}$$

This is equivalent to maximizing:

$$u(w - s^* - s) + \delta u(sR + s^*R)$$

First-order condition:

$$u'(w - s^* - s) = R\delta\, u'(sR + s^*R)$$

Recall that the socially optimal savings s^* satisfies:

$$u'(w - s^*) = R\delta\, u'(s^*R)$$

Comparing these conditions, we obtain:

$$\boxed{s = 0}$$

Conclusion: For rational individuals, mandatory pension savings perfectly crowd out private savings. Total savings remain at the optimal level s^*.

- For a **myopic individual** (who only cares about current consumption):

$$\max_s \; u(c_1) \quad \text{s.t.} \quad c_1 + s + s^* = w, \quad c_2 = sR + s^*R$$

This simplifies to maximizing $u(w - s^* - s)$.

  - If borrowing is not allowed (no borrowing against future pension income):$s = 0$
  - If borrowing is allowed: Banks know the individual will receive s^*R in period 2 and may lend against this future income. The myopic individual would borrow as much as possible: $s = -s^*$
  - In reality, it is typically not possible to borrow against future pension benefits. Therefore, the relevant case is $s = 0$.

5.2.5 Summary

Key Takeaways

- **Socially optimal savings:** s^*
- **Without a pension system:**
  - Rational individuals save s^*
  - Myopic individuals save 0
- **With a mandatory pension system (contribution s^*):**
  - Rational individuals: Private savings = 0, pension savings = s^*, total savings = s^*
  - Myopic individuals (no borrowing): Private savings = 0, pension savings = s^*, total savings = s^*
  - Myopic individuals (with borrowing): Private savings = $-s^*$, pension savings = s^*, total savings = 0 (but borrowing is usually not feasible)
- Conclusion: A mandatory pension system **does not change total savings for rational individuals** (crowding out), but it **forces myopic individuals to save**, achieving the socially optimal level.

5.2.6 The Impact on Labor Supply

Introducing labor supply into pervious two-period OLG model, we can get the following setting:

- In period 1, individuals choose labor supply $l_1, \ 0 \le l_1 \le 1$ (fraction of time worked).
- Consumption in period 1:
$$c_1 = (1-\tau)wl_1 - s$$
where τ is the pension contribution rate, w is the wage rate, and s is private savings.
- Question: What essential assumption is missing?
  - We need to specify the *disutility of labor* (value of leisure).
  - Let $e_1 = 1 - l_1$ be leisure time in period 1.
  - Utility from leisure is given by $h(e_1)$, with $h' > 0$ and $h'' < 0$.
- Period 1 utility becomes:
$$u(c_1) + h(e_1)$$

The rational individual maximizes lifetime utility:

$$\max_{l_1, s} \ u(c_1) + h(1-l_1) + \delta u(c_2)$$

subject to:

$$c_1 = (1-\tau)wl_1 - s$$
$$c_2 = R(s + \tau w l_1)$$

Eliminating s, we obtain Intertemporal Budget Constraint:

$$c_1 + \frac{c_2}{R} = wl_1$$

Key observation: The budget constraint is **independent of** τ! The pension system is equivalent to forced savings that are fully offset by private savings.
For rational individuals, the pension system does not affect labor supply. The optimal l_1 is determined solely by the trade-off between consumption and leisure, independent of τ.

A myopic individual only cares about current period utility:

$$\max_{l_1, s} \ u(c_1) + h(1-l_1)$$

subject to the same constraints and $s \ge 0$ (borrowing constraint).
It is easy to see that the optimal private savings is $s = 0$ (myopic individuals do not save). Substituting $s = 0$ into the constraints:

$$c_1 = (1-\tau)wl_1, \quad c_2 = \tau w l_1 R$$

The myopic individual's problem reduces to:

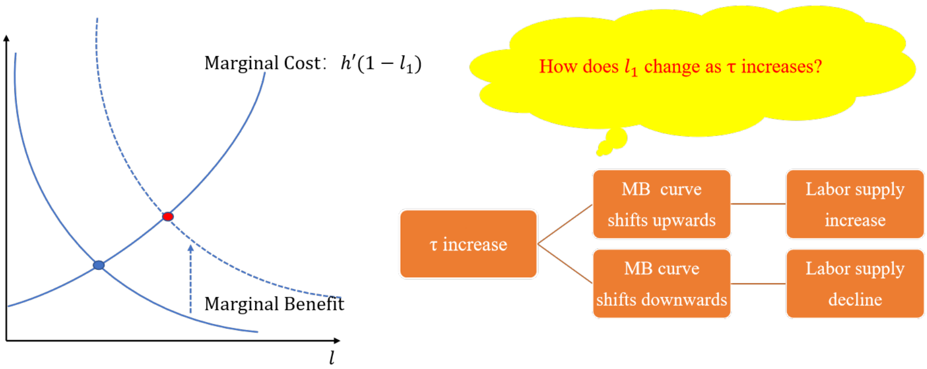
$$\max_{l_1} \ u((1-\tau)wl_1) + h(1-l_1)$$

First-order condition:

$$\underbrace{(1-\tau)w\,u'((1-\tau)wl_1)}_{\text{Marginal benefit}} = \underbrace{h'(1-l_1)}_{\text{Marginal cost}}$$

The first-order condition determines labor supply l_1 as a function of τ:

$$(1-\tau)w\,u'\big((1-\tau)wl_1\big) = h'(1-l_1)$$



5.5: Impact on Labor Supply: Myopic Individuals

How does the marginal benefit curve shift with τ?

$$\frac{\partial \text{MB}}{\partial \tau} = \frac{\partial}{\partial \tau}\left[(1-\tau)w\,u'((1-\tau)wl_1)\right]$$

$$= -wu'(c_1) + (1-\tau)w\,u''(c_1)\cdot(-wl_1)$$

$$= -wu'(c_1) - (1-\tau)w^2 l_1 u''(c_1)$$

$$= w\left[-u'(c_1) - c_1 u''(c_1)\right]$$

Key Insight

The sign of $\frac{\partial \text{MB}}{\partial \tau}$ depends on the curvature of the utility function:

$$\frac{\partial \text{MB}}{\partial \tau} = w\left[-u'(c_1) - c_1 u''(c_1)\right]$$

- If $-u'(c_1) - c_1u''(c_1) > 0$, then $\frac{\partial \text{MB}}{\partial \tau} > 0$: higher τ *increases* marginal benefit of working $\Rightarrow$ labor supply increases.
- If $-u'(c_1) - c_1u''(c_1) < 0$, then $\frac{\partial \text{MB}}{\partial \tau} < 0$: higher τ *decreases* marginal benefit of working $\Rightarrow$ labor supply decreases.

Therefore, the effect of a mandatory pension on labor supply for myopic individuals is theoretically ambiguous.

Summary

- Rational individuals: Pension system does not affect labor supply (full crowding out via private savings).
- Myopic individuals:
  - They do not save privately ($s = 0$).
  - Labor supply is determined by the trade-off between after-tax consumption and leisure.
  - The effect of an increase in τ on labor supply is ambiguous, depending on the individual's risk aversion and the curvature of the utility function.

An Intuitive Explanation

Recall the myopic individual's problem:

$$\max_{l_1} \; u((1-\tau)wl_1) + h(1-l_1)$$

Alternatively, expressing in terms of consumption c_1 and leisure $e_1 = 1 - l_1$:

$$\max_{c_1, e_1} \; u(c_1) + h(e_1) \quad \text{s.t.} \quad c_1 = (1-\tau)w(1-e_1)$$

Rewriting the budget constraint:

$$\frac{1}{(1-\tau)w} c_1 + e_1 = 1$$

Key Insight

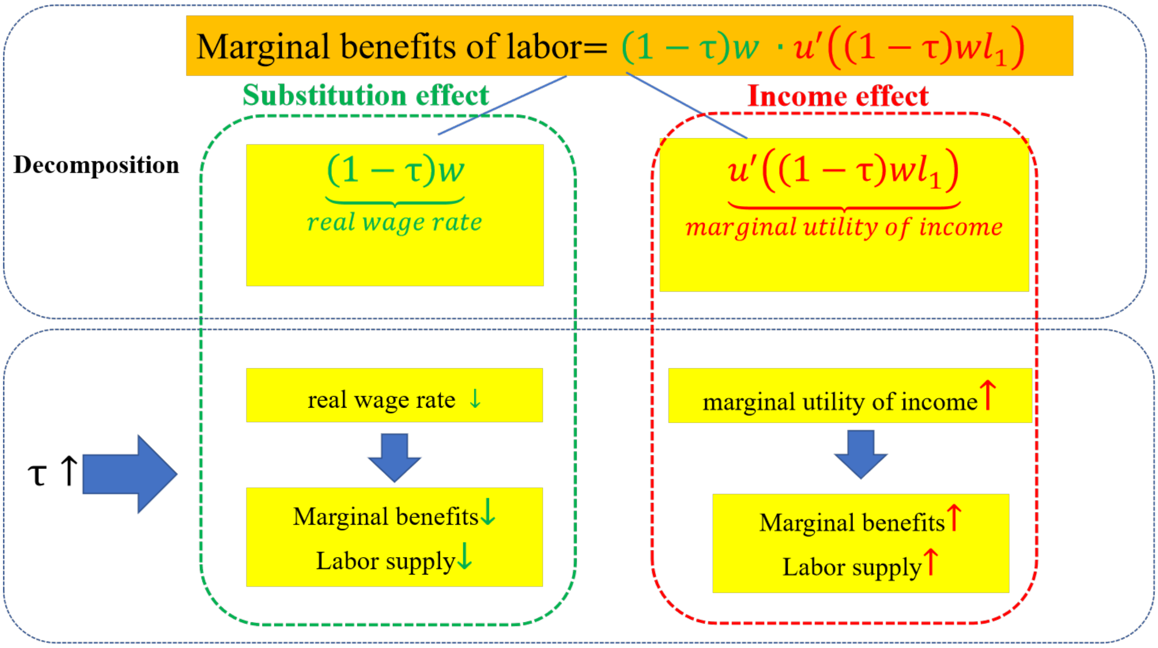
This is a standard consumer choice problem between two goods is: consumption c_1 (price $p_c = \frac{1}{(1-\tau)w}$) and leisure e_1 (price $p_e = 1$), with total endowment of time normalized to 1.

How Does an Increase in τ Affect Labor Supply? If your daily wage was originally ¥ 500 and now it has dropped to ¥ 50 , would you be inclined to increase or decrease your labor supply?
When τ increases:

- The after-tax wage $(1-\tau)w$ decreases.
- This means the price of consumption $p_c = \frac{1}{(1-\tau)w}$ increases.
- The price of leisure $p_e = 1$ remains unchanged.

Recall from microeconomics: when the price of a good increases,

- **Substitution effect:** The relatively cheaper good (leisure) is substituted for the more expensive good (consumption) $\Rightarrow$ leisure increases, labor supply decreases.
- **Income effect:** The increase in price reduces real income $\Rightarrow$ demand for both goods may decrease (if normal goods) $\Rightarrow$ leisure decreases, labor supply increases.



5.6: Substitution and Income Effects of an Increase in τ

Summary of Effects

- Substitution effect: $\tau \uparrow \Rightarrow p_c \uparrow \Rightarrow e_1 \uparrow \Rightarrow l_1 \downarrow$
- Income effect: $\tau \uparrow \Rightarrow$ real income $\downarrow \Rightarrow e_1 \downarrow \Rightarrow l_1 \uparrow$

Conclusion

The net effect of an increase in the pension contribution rate τ on labor supply is theoretically ambiguous:

$$\text{Total effect} = \text{Substitution effect}(- \text{ for } l_1) + \text{Income effect}(+ \text{ for } l_1)$$

Whether labor supply increases or decreases depends on which effect dominates. This mirrors the standard result from labor economics: the effect of a wage change on labor supply is ambiguous due to opposing income and substitution effects.

5.3 Empirical Studies on Social Insurance

5.3.1 Wang et al. JPubE 2024: NCMS and Entrepreneurship

Highlights

- Public health insurance substantially boosts rural entrepreneurship in China.
- It raises household's participation and hours in entrepreneurial works.
- Impact is U-shaped concerning pre-program health risks.
- In aggregate, it fosters the growth of rural smallholder businesses.
- Public health insurance facilitates rural financial decision-making beyond healthcare.

- **Research Topic:** The impact of the New Cooperative Medical Scheme (NCMS) on entrepreneurship
  - Individuals are risk-averse.
  - In the presence of health risk, without health insurance, individuals are more inclined to choose low-risk jobs to avoid introducing additional risk through entrepreneurial activities.
  - With health insurance, the willingness to become an entrepreneur increases.

- **Data Source**
  The CHNS provides a longitudinal survey dataset with a nationally representative sample. It uses a multistage random cluster process, drawing more than 7,200 households with over 30,000 individuals from 15 provinces, autonomous regions, and municipalities.

  The survey includes rich individual-level information on demographics, health, time use, and education; household-level data on income and family structure; and community-level characteristics such as local market conditions, public infrastructure for health and social services, all of which are analyzed in the study.

- **Sample Selection Criteria**

1. Only retain samples with rural household registration (rural Hukou).
2. Only retain samples where the household head and spouse remained unchanged throughout the survey period.
3. Set an age lower bound: only retain individuals who were already over 18 years old when first appearing in the survey.
4. Exclude observations surveyed in the same year that NRCMS was implemented.

Overall, the analytical sample contains 11,354 observations from 2,041 households across 119 rural communities in 36 counties.

Identification Based on Micro-Data: DID Method

$$Y_{hct} = \beta_1 NRCMS_{ct} + \beta_2 X_{ht} + \beta_3 \Theta_h \times f(t) + \beta_4 \Gamma_c \times f(t) + \beta_5 Z_{ct} + \delta_h + \gamma_t + \epsilon_{hct}$$

- Y_{hct}: Outcome variable for household h in county c in year t
- $NRCMS_{ct}$: Whether NCMS was implemented in county c by year t
- X_{ht}: Household characteristics
- $\Theta_h \times f(t)$: Household characteristics $\times$ time trend
- $\Gamma_c \times f(t)$: County characteristics $\times$ time trend
- Z_{ct}: County characteristics
- δ_h: Household fixed effects
- γ_t: Year fixed effects

Occupation Classification

- High-risk occupations:
  - Self-employment in small businesses (entrepreneurship)
  - Paid work in the private sector
- Low-risk occupations:
  - Agricultural work
  - Paid work in the public sector

Table 2
Effects of the NRCMS on household head's labor supply in high-risk occupations.

	(1)	(2)	(3)	(4)	(5)	(6)
	Participation (extensive margin)			Log hours worked (intensive margin)		
NRCMS	0.096***	0.093***	$\boxed{0.083^{***}}$	0.700***	0.677***	$\boxed{0.593^{***}}$
	(0.030)	(0.030)	$\boxed{(0.028)}$	(0.214)	(0.216)	$\boxed{(0.202)}$
Household and year FE	✓	✓	✓	✓	✓	✓
Household controls and trends		✓	✓		✓	✓
County trends			✓			✓
R-squared	0.015	0.019	0.025	0.016	0.021	0.027
No. of households	2,041	2,041	2,041	2,041	2,041	2,041
Observations	11,354	11,354	11,354	11,354	11,354	11,354
Pre-program outcome mean	0.253	0.253	0.253	540	540	540

Notes: This table reports the estimated effect of the NRCMS on participation in high-risk work of household heads in rural China. The regression model is specified in Eq. (1). Dependent variables are the participation dummy in Columns 1–3 and the log annual working hours in Columns 4–6. Household controls include the age and age squared of the household head. Household trends include sets of household pre-determined characteristics, including an indicator of the head's being female and indicators for the highest educational attainment for the head and the spouse, interacting with quadratic time trends. County trends include quartile dummies of various pre-program county characteristics interacted with quadratic time trends. The set of county characteristics includes average household income, share of rural population, and index for local government's fiscal capacity (index of transportation infrastructure and index of public services). See Online Appendix Table B3 for more details. Pre-program outcome mean measured in level terms is listed at the bottom row. The outcome means in Columns 4–6 is measured in hours. Robust standard errors clustered at the county level are reported in parentheses. *** <0.01, ** <0.05, * <0.1.

The NRCMS has increased the participation rate of high-risk work by 8.3 percentage points and increased the participation time of high-risk work by 59.3%.

5.7: Regression results based on micro-household data

5.3.2 García-Miralles and Leganza JPubE 2024: Joint Retirement

Highlights

- Many couples retire together when one spouse reaches pension eligibility age.
- We uncover the pathways these couples take to retire together.
- Age differences are crucial: older spouses wait for their partners to be eligible too.
- We also highlight roles for gender and relative earnings between spouses.
- A reform that increased the pension eligibility age generated similar spousal spillovers.

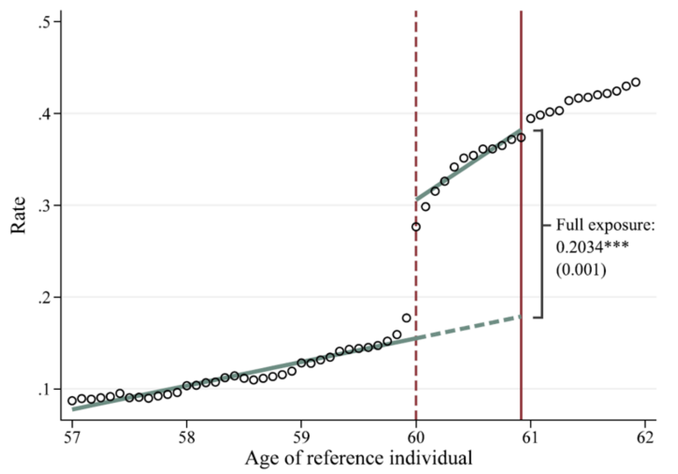
- **Institution Background:** In the Danish retirement system, the Old Age Pension (OAP) provides universal retirement income security at old ages, and the Voluntary Early Retirement Pension (VERP) provides early retirement benefits for those who choose to participate in the program.
- **Data:** Danish population-wide administrative databases, 1986–2014

- **Variable Definitions:**
  - *Retirement:* Defined as the state when an individual stops receiving labor income.
  - *Pension receipt:* Defined as the state when an individual receives any form of pension (including VERP and OAP) in a given year.
  - *Labor market income:* The individual's total taxable labor market income.
  - *Wealth:* Includes total assets of the individual and household (excluding housing and pension wealth).
- **Sample Selection:**
  - *Couple definition:* Couples in the sample include married, registered partners, or cohabiting spouses. To avoid endogeneity issues caused by changes in marital status near the pension eligibility age, we determine marital status when both spouses are under 60 and maintain this status in subsequent observations.
  - *Age restriction:* The analysis sample is limited to couples with an age difference of no more than 8 years to reduce heterogeneity caused by large age gaps.
  - *Dual-earner couples:* Select spouses with at least one year of labor income between ages 55 and 59. Exclude self-employed individuals and those receiving disability benefits.
  - *Observation window:* To analyze the impact of pension eligibility age on retirement behavior, we select samples where the individual is between ages 57 and 60.

Identification Strategy: Effect of Reaching Age 60 on Own Retirement Decision

$$y_{it} = \alpha + \beta_1(age_{it} - 60) + \beta_2\{age_{it} \geq 60\} + \beta_3\{age_{it} \geq 60\} \cdot (age_{it} - 60)$$
$$+ \sum_{c=1991}^{2013} \kappa_c D_c + \epsilon_{it}$$

(a) Retirement

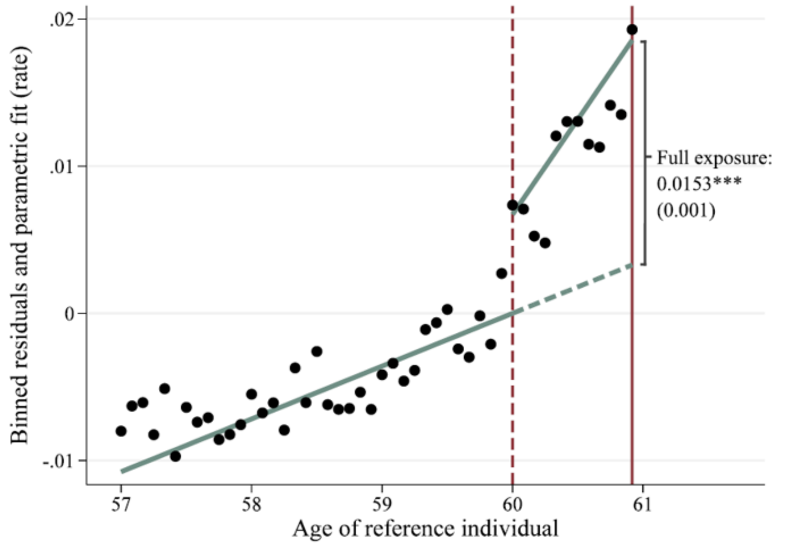


5.8: Own Retirement Decision

Identification Strategy: Effect of Reaching Age 60 on Spouse's Retirement Decision

$$
\begin{aligned}
y^s_{it} &= \alpha + \beta_1(age_{it} - 60) + \beta_2\{age_{it} \geq 60\} + \beta_3\{age_{it} \geq 60\} \cdot (age_{it} - 60) \\
&+ \sum_{a=49}^{69} \delta_a \cdot D^s_a + \sum_{a=49}^{69} \gamma_a \cdot D^s_a \cdot D_g + \sum_{c=1991}^{2013} \kappa_c \cdot D_c + \varepsilon_{it}
\end{aligned}
$$

(a) Spouse Retirement



5.9: Spouse's Retirement Decision

Following table shows the regression result.

Table 1
The effect of reaching pension eligibility age.

	Retirement	Claiming	Earnings
Reference individual	0.2034***	0.3496***	−8,642***
	(0.001)	(0.001)	(69)
Spouse	0.0153***	0.0120***	−848***
	(0.001)	(0.001)	(61)
Scaled effect	0.0750***	0.0344***	0.0981***
	(0.007)	(0.003)	(0.012)
N. of clusters	367,585	367,585	367,585
Observations	2,206,044	2,206,044	2,206,044

Reaching the early retirement age increases an individual's retirement rate by 20.34 percentage points and increases the spouse's retirement rate by 1.53 percentage points.

For every 100 individuals who retire upon reaching the early retirement age, 7.5 spouses also retire jointly.

5.10: Baseline Result

Table 5
Heterogeneity in the effect of reaching pension eligibility age on retirement by age difference, gender, and relative earnings.

Spouse	Age difference		Gender		Relative earnings	
	Old (1)	Young (w) (2)	Male (3)	Female (w) (4)	Primary (5)	Secondary (w) (6)
Reference individual	0.2562***	0.1893***	0.2668***	0.1788***	0.2423***	0.1975***
	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)
Spouse	0.026***	0.007***	0.020***	0.025***	0.020***	0.021***
	(0.002)	(0.001)	(0.002)	(0.003)	(0.001)	(0.002)
Scaled effect	0.0994***	0.0414***	0.0745***	0.139***	0.0747***	0.107***
	(0.010)	(0.010)	(0.009)	(0.018)	(0.009)	(0.014)
N. of clusters	297,686	334,966	302,589	330,172	300,312	332,422
Observations	1,038,096	1,167,948	1,054,359	1,151,685	1,056,592	1,149,452

Notes: This table reports the effect of the reference individuals reaching pension eligibility age on their own retirement and on their spouses' retirement, distinguishing heterogeneous responses by age differences within the couple, by gender, and by relative earnings between partners. For each heterogeneity cut, the columns marked with (w) are reweighted to be comparable to the remaining sample. For example, columns (1) and (2) show a split by age difference within the couple, and column (2) is reweighted to have the same joint distribution of gender and relative income as column (1). Likewise, column (4) is reweighted to have the same joint distribution of age difference and relative income as column (3), and column (6) is reweighted to have the same joint distribution of age differences and gender as column (5). The first row reports the full-exposure effect of pension eligibility on own retirement. The second row reports the full-exposure effect on spouses of their partners being eligible for retirement pension. The third row reports the scaled effect resulting from diving the spouse full-exposure effect by the reference individual full-exposure effect. Robust standard errors in parentheses, clustered at the couple level. Bootstrapped standard errors for scaled effects. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

5.11: Heterogeneity Analysis

Heterogeneity analysis shows that:

- Older spouses have a larger joint retirement effect.
- Women are more likely to retire jointly with their husbands.
- Lower-income individuals are more likely to retire jointly.

5.3.3 Farré and González JPubB 2019: Fertility Timing and Subsequent Childbearing

Highlights

- Two weeks of paid paternity leave in Spain led to delays in subsequent fertility.
- Parents (just) entitled to the new leave took longer to have another child.
- Eligible couples were less likely to have another child within the next six years.
- We provide evidence in support of two potentially complementary channels.

- **Institutional Background**
  - In 2007, Spain introduced a 13-day paternity leave policy.
  - The policy applies to children born on or after March 24, 2007.
- **Potential Channels**: How Paternity Leave May Affect Fertility
  - Channel 1: Maternal employment $\uparrow \Rightarrow$ Opportunity cost of childbearing $\uparrow$ $\Rightarrow$ Fertility $\downarrow$
  - Channel 2: Paternal childcare time $\uparrow \Rightarrow$ Fathers become more aware of the high costs of parenting $\Rightarrow$ Willingness to have children $\downarrow \Rightarrow$ Fertility $\downarrow$

Identification Strategy: Regression Discontinuity

- **Sample:** Families giving birth within a few weeks before and after March 24

$$Y_{id} = \alpha + \beta T_{id} + \gamma_1 f(d) + \gamma_2 f(d) T_{id} + \delta X_{id} + \epsilon_{id}$$

- i: family, d: date of childbirth
- Y_{id}: time interval until next birth; whether the family has another child within 2, 4, or 6 years
- T_{id}: whether the child was born after March 24

Table 1
Effect of paternity leave on birth spacing (days to subsequent birth).

Window	Opt. band.	±7 days	±21 days	±56 days	±77 days	±90 days	±90 d. (donut)
Panel A. All ages							
Paternity leave	27.1*	20.7	27.8*	20.5**	16.0**	32.5***	38.2**
	(13.6)	(11.9)	(13.8)	(8.8)	(7.2)	(10.4)	(15.3)
Mean	1248	1251	1249	1250	1249	1249	1249
Effect as % of mean	2.2%	1.7%	2.2%	1.6%	1.3%	2.6%	3.1%
Linear trends	Y	N	Y	Y	Y	Y	Y
Quadratic trends	N	N	N	N	N	Y	Y
Day of the week	Y	N	Y	Y	Y	Y	Y
N	18,329	6473	19,246	51,628	71,341	83,480	77,007

After paternity leave, the birth spacing increases.

5.12: Effect on birth spacing

Identification Strategy: Difference-in-Differences to Address Seasonal Fluctuations

- **Sample:** Families giving birth around March 24 in 2006 (control group), 2007 (treatment group), and 2008 (control group)

$$Y_{idt} = \alpha + \beta_1 T_{idt} + \beta_2 T_{idt} I_{2007} + \gamma f(\delta) + \theta X_{id} + \mu_t + \epsilon_{id}$$

- T_{idt}: whether after March 24
- I_{2007}: whether year 2007

Appendix Table 4
Effect of paternity leave on birth spacing (days to subsequent birth), difference-in-difference specification.

Window	All ages				Older mothers (>30)	
	±77 days	±77 days	Full year	Full year	±77 days	Full year
	7.61***	8.02*	6.97**	6.98**	15.47**	12.39**
Paternity leave	(4.65)	(4.70)	(3.51)	(3.47)	(6.25)	(4.86)
Mean	1251	1251	1221	1221	1194	1165
Effect as % of mean	0.6%	0.6%	0.6%	0.6%	1.3%	1.1%
N	204,756	204,756	506,936	506,936	100,857	238,058
Day of the week	N	Y	N	Y	N	Y
Linear trend	Y	Y	Y	Y	Y	Y
Cubic trend	N	Y	N	Y	N	Y

5.13: DID result

Table 2
Effect of paternity leave on subsequent fertility.

Window	Opt. band.	±7 days	±21 days	±56 days	±77 days	±90 days	±90 d. (donut)
Panel A. All ages							
Two years							
Paternity leave	−0.0097*	−0.0068***	−0.0097*	−0.0050	−0.0050**	−0.0084**	−0.0086*
	(0.0053)	(0.0048)	(0.0053)	(0.0031)	(0.0025)	(0.0036)	(0.0049)
Average	0.066	0.064	0.066	0.067	0.067	0.067	0.067
Coeff./average	−14.8%	−10.6%	−14.8%	−7.5%	−7.2%	−12.6%	−12.9%
Four years							
Paternity leave	−0.0154**	−0.0066	−0.0120*	−0.0018	−0.0059	−0.0101*	−0.0118
	(0.0068)	(0.0057)	(0.0061)	(0.0044)	(0.0038)	(0.0054)	(0.0085)
Average	0.239	0.238	0.239	0.239	0.239	0.240	0.240
Coeff./average	−6.4%	−2.8%	−5.0%	−0.8%	−2.5%	−4.2%	−4.9%
Six years							
Paternity leave	−0.0179*	−0.0093	−0.0129	−0.0023	−0.0071*	−0.0116*	−0.0115
	(0.0091)	(0.0070)	(0.0085)	(0.0050)	(0.0043)	(0.0059)	(0.0108)
Average	0.339	0.336	0.338	0.338	0.337	0.339	0.339
Coeff./average	−5.3%	−2.8%	−3.8%	−0.7%	−2.1%	−3.4%	−3.4%
Linear trend	Y	N	Y	Y	Y	Y	Y
Quadratic trend	N	N	N	N	N	Y	Y
Day of the week	Y	N	Y	Y	Y	Y	Y
N	(varying)	18,174	53,693	144,055	199,558	232,484	214,310

After paternity leave, the probability of having another child decreases.

5.14: Effect on subsequent fertility

Mechanism 1: Impact on Labor Market

Table 3
Effect of paternity leave on labor market outcomes.

	Father			Mother		
	±3 months	±6 months	±9 months	±3 months	±6 months	±9 months
	−0.003	0.001	0.010	0.040***	0.038***	0.025
Working after 6 months	(0.006)	(0.009)	(0.012)	(0.009)	(0.013)	(0.017)
	−0.003	0.008	−0.001	0.025***	0.031**	0.028*
Working after 12 m.	(0.007)	(0.010)	(0.012)	(0.009)	(0.013)	(0.017)
	−0.008	0.008	0.008	0.011	0.027**	0.036**
Working after 24 m.	(0.008)	(0.012)	(0.015)	(0.009)	(0.014)	(0.017)
	0.000	0.000	0.000	−0.019***	−0.024***	−0.018*
On leave after 6 m.	(0.001)	(0.001)	(0.001)	(0.005)	(0.007)	(0.010)
	0.001	0.001	0.001	−0.021***	−0.025***	−0.020**
On leave after 12 m.	(0.001)	(0.001)	(0.002)	(0.005)	(0.008)	(0.010)
	0.002	0.001	0.002	−0.018***	−0.022***	−0.018*
On leave after 24 m.	(0.001)	(0.002)	(0.002)	(0.005)	(0.008)	(0.010)
	−0.002	0.000	0.002	0.015**	0.024**	0.016
Part-time emp. after 6 m.	(0.005)	(0.008)	(0.010)	(0.007)	(0.011)	(0.014)
	−0.002	−0.001	−0.009	0.010	0.020*	0.021
Part-time emp. after 12 m.	(0.006)	(0.008)	(0.010)	(0.008)	(0.012)	(0.015)
	−0.009	−0.008	−0.013	−0.002	0.011	0.012
Part-time emp. after 24 m.	(0.005)	(0.008)	(0.010)	(0.008)	(0.012)	(0.015)
	−42.09	−89.48	−20.64	219.98***	112.80	30.50
Earnings after 6 m.	(84.85)	(121.71)	(154.67)	(68.34)	(99.56)	(127.46)
	23.64	−105.10	−78.77	551.33***	381.37*	250.98
Earnings after 12 m.	(169.06)	(242.69)	(308.67)	(138.18)	(200.52)	(256.68)
	−138.54	−10.07	40.38	380.17**	389.89*	495.92*
Earnings after 24 m.	(184.35)	(6.17)	(336.11)	(155.91)	(226.22)	(289.84)
N	7591	15,626	23,477	7665	15,899	23,853
Linear trend	N	Y	Y	N	Y	Y
Quadratic trend	N	N	Y	N	N	Y

5.15: Effect of paternity leave on labor market outcomes

- **Key finding: Paternity leave increases employment opportunities for mothers after childbirth.**

Mechanism 2: Impact on Fathers' Fertility Preferences

Table 4
Effect of paternity leave on the time-use of fathers and mothers (DiD, ±3 months).

Min. per day on:	Fathers OLS	Fathers Tobit	Fathers OLS	Fathers Tobit	Mothers OLS	Mothers Tobit	Mothers OLS	Mothers Tobit
Childcare	45.529 (28.283)	58.320* (31.468)	52.606 (32.769)	71.948** (36.168)	−13.080 (33.060)	−10.031*** (32.648)	−15.146 (40.640)	−13.190 (38.675)
Housework	−6.229 (28.838)	3.392 (34.730)	10.005 (34.470)	23.773 (40.413)	42.430 (27.528)	44.706 (27.368)	42.636 (37.361)	45.707 (36.517)
Work	29.290 (70.189)	39.975 (118.990)	−35.763 (78.922)	−104.885 (134.664)	25.624 (50.139)	54.328 (117.281)	13.452 (68.153)	−56.846 (158.584)
Residual	−69.095 (59.404)	−69.095 (56.345)	−27.825 (69.893)	−27.825 (65.419)	−54.940 (40.200)	−54.940 (38.286)	−45.489 (54.518)	−45.489 (50.815)
N	290	290	235	235	313	313	222	222
Age range	16–55	16–55	30–55	30–55	16–55	16–55	30–55	30–55

5.16: Effect of paternity leave on the time-us

- **Key finding: Paternity leave increases father's parenting time and reduces mother's parenting time.**

Table 5
Changes in desired fertility, men vs. women.

	Basic spec.	LM controls	Full spec.	Poisson
Men*2011	−0.1995* (0.1134)	−0.2203* (0.1146)	−0.2214** (0.1131)	−0.1086** (0.0551)
Men	0.0552*** (0.0762)	0.0546 (0.0777)	0.1955* (0.1089)	0.0988* (0.0556)
N	871	871	868	868
Age polynomial	Y	Y	Y	Y
Educ. and emp. controls	N	Y	Y	Y
Married and n. children controls	N	N	Y	Y

5.17: Changes in desired fertility, men vs. women.

- Sample: 2001, 2006, 2011
- **Key finding: Paternity leave reduces fathers' desired fertility.**

Mechanism 3: Impact on Marital Stability

Appendix Table 8
Effect of paternity leave on separation and divorce.

	Divorced ±3 months	Divorced ±6 months	Divorced ±9 months	Not cohabiting ±3 months	Not cohabiting ±6 months	Not cohabiting ±9 months
2008	0.009 (0.007)	0.003 (0.009)	0.008 (0.011)	0.013 (0.009)	0.015 (0.014)	0.026 (0.017)
2009	0.004 (0.007)	0.017* (0.009)	0.024** (0.012)	−0.003 (0.010)	0.014 (0.014)	0.011 (0.017)
2010	−0.008 (0.006)	−0.025** (0.010)	−0.028** (0.013)	−0.022** (0.009)	−0.036*** (0.013)	−0.035** (0.017)
2011	−0.003 (0.007)	−0.014 (0.010)	−0.015 (0.013)	−0.010 (0.010)	−0.025* (0.014)	−0.029* (0.017)
2012	−0.000 (0.008)	0.004 (0.012)	0.014 (0.015)	−0.001 (0.011)	−0.002 (0.016)	0.003 (0.020)
2013	0.005	0.010	0.009	0.007	0.020	0.006

5.18: Effect of paternity leave on separation and divorce.

- **Key finding: This mechanism is not significant. The divorce rate of families who enjoyed paternity leave decreased in 2010, but the impact has since tended towards 0, which means paternity leave has little impact on marital stability.**

Chapter 6 Tax Incidence

This chapter is divided into two main sections. *Partial equilibrium analysis* examines tax incidence within a single market, using examples, general formulas, elasticity concepts, and empirical studies. *General equilibrium analysis* then expands the scope to multiple interconnected markets, employing the Harberger model to analyze how taxes affect all prices and the distribution of burden between labor and capital.

6.1 Partial Equilibrium

Reviewing of Principles of Public Finance Course, we learned that taxpayer and tax bearer are not the same concept. The tax bearer is the unit or individual who ultimately bears the tax burden, and the tax bearer is not necessarily the taxpayer specified in tax law. In this way, we can define the following concepts.

Definition 6.1.1: Tax Incidence

The person who ultimately bears the tax burden cannot shift the tax further, becoming the endpoint of the tax shifting process. This final bearer of the tax burden is also called the tax incidence.

Definition 6.1.2: Tax Shifting

Refers to an economic process where the taxpayer transfers the tax they have paid to others to bear.

- Relationship between the difficulty of tax shifting and the price elasticity of supply and demand

6.1.1 Tax Incidence

Tax Incidence: Studies the impact of tax policies on various prices and on the social welfare of different economic agents.

- **Statutory incidence**: Similar to the taxpayer
- **Economic incidence**: Similar to the bearer of the tax burden

e.g. 9: If a \$1 excise tax is imposed on each pack of cigarettes, what impacts will it have?

- How does it affect the retail price of cigarettes, thereby influencing smokers' behavior?
- How does it affect producer profits and shareholder equity?
- How does it affect the income of workers in tobacco companies?
- How does it affect the income of farmers growing tobacco?

- **Importance of Tax Incidence**: It is the first step in policy evaluation.

6.1.2 Partial Equilibrium Analysis

- Analyzes a single market
- Usually assumes a perfectly competitive market, using supply and demand curves

e.g. 10: Example Question

In a perfectly competitive market, the demand function for cigarettes is $Q = 4000 - 200P$, and the supply function is $Q = -500 + 100P$. If a \$3 per-unit tax is imposed on each pack of cigarettes, the tax burden borne by the producer per pack is \$_____ , and the tax burden borne by the consumer is \$_____.

Pre-tax Equilibrium

$$Q = 4000 - 200P = -500 + 100P$$

$$\mathbf{P = 15,\ Q = 1000}$$

Assume a \$3 per-unit tax is levied on the firm

- Let P be the price paid by consumers, then the firm receives $P - 3$

$$Q = 4000 - 200P = -500 + 100(P - 3)$$

$$\mathbf{P = 16,\ Q = 800}$$

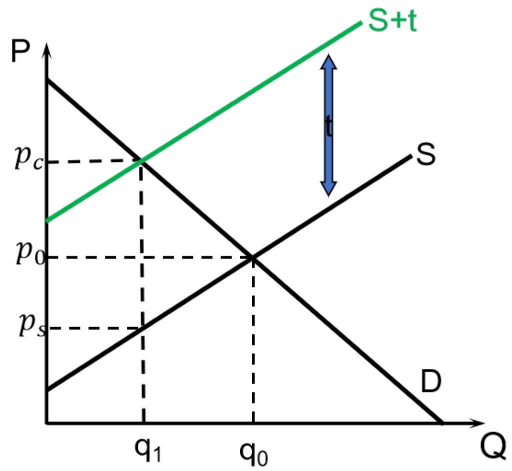
- Consumers pay a total of \$16, bearing \$1 of the tax per pack
- Firms actually receive \$13, bearing \$2 of the tax per pack

Assume a \$3 per-unit tax is levied on consumers

- Let P be the total cost paid by consumers (price to the firm + tax), then the firm receives $P - 3$. This is the same as the case where the tax is levied on the firm.

Key Insight 1

Tax incidence is independent of whom the tax is levied on!



- Before tax: Equilibrium price is p_0.
- After imposing a per-unit tax t:
  - The price paid by consumers is p_c.
  - The price received by producers is p_s.
- The tax burden is shared by producers and consumers.

6.1.2.1 General Case for Partial Equilibrium

Demand function:

$$Q = D(P)$$

Supply function:

$$Q = S(P)$$

- Impose a per-unit tax of t yuan on each unit of the good
  - How does the equilibrium price change?
  - What is tax incidence related to?
- Solution:
  - Let P be the total cost paid by the consumer, then $P - t$ is the price received by the firm.
  - At equilibrium:

$$D(P) = S(P - t)$$

  - Need to solve for $\frac{dP}{dt}$.

Totally differentiate $D(P) = S(P - t)$:

$$D'(P)dP = S'(P - t)(dP - dt)$$

Solve for $\frac{dP}{dt}$:

$$\frac{dP}{dt} = \frac{S'(P - t)}{S'(P - t) - D'(P)}$$

Since $S'(P - t) > 0$ and $D'(P) < 0$:

$$0 < \frac{dP}{dt} < 1$$

Key Insight 2

In general, the tax burden is shared by consumers and firms.

Introducing Price Elasticities

- Price elasticity of demand:

$$\varepsilon_D = -\frac{dQ}{dP} \cdot \frac{P}{Q} = -\frac{dD(P)}{dP} \cdot \frac{P}{Q} = -D'(P)\frac{P}{Q}$$

- Price elasticity of supply:

$$\varepsilon_S = \frac{dQ}{dP} \cdot \frac{P}{Q} = \frac{dS(P)}{dP} \cdot \frac{P}{Q} = S'(P)\frac{P}{Q}$$

- Note: At equilibrium, $Q = S = D$.

6.1.2.2 Relationship between Tax Incidence and Price Elasticities

$$\frac{dP}{dt} = \frac{S'(P - t)}{S'(P - t) - D'(P)} = \frac{S'(P - t)\frac{P-t}{S}}{S'(P - t)\frac{P-t}{S} - D'(P)\frac{P-t}{S}\frac{P-t}{P}}$$

When t approaches 0, $\frac{P-t}{P}$ approaches 1.

$$\frac{dP}{dt} = \frac{\varepsilon_S}{\varepsilon_S + \varepsilon_D}$$

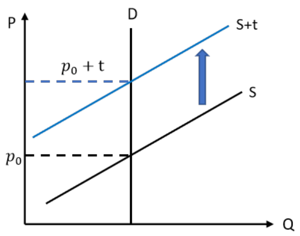
- Supply Price Elasticity ε_S:
  - Larger $\varepsilon_S \Rightarrow$ Larger $\frac{dP}{dt} \Rightarrow$ Price increases more after tax $\Rightarrow$ Consumers bear more tax burden.
  - If supply is perfectly elastic ($\varepsilon_S = \infty$): $\frac{dP}{dt} = 1 \Rightarrow$ Tax burden falls entirely on consumers.
  - If supply is perfectly inelastic ($\varepsilon_S = 0$): $\frac{dP}{dt} = 0 \Rightarrow$ Tax burden falls entirely on producers.
- Demand Price Elasticity ε_D:
  - Larger $\varepsilon_D \Rightarrow$ Smaller $\frac{dP}{dt} \Rightarrow$ Price increases less after tax $\Rightarrow$ Producers bear more tax burden.

- If demand price elasticity is ∞ ($\varepsilon_D = \infty$): $\frac{dP}{dt} = 0 \Rightarrow$ Tax burden falls entirely on consumers.
- If demand price elasticity is infinite($\varepsilon_D = 0$): $\frac{dP}{dt} = 1 \Rightarrow$ Tax burden falls entirely on producers.

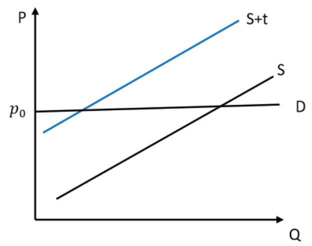
Key Insight 3

Those with lower price elasticity bear more tax burden

- High price elasticity of demand $\Rightarrow$ Demand is sensitive to price$\Rightarrow$A price increase leads to a large drop in demand$\Rightarrow$Difficulty in shifting the tax$\Rightarrow$The tax burden falls more on producers.

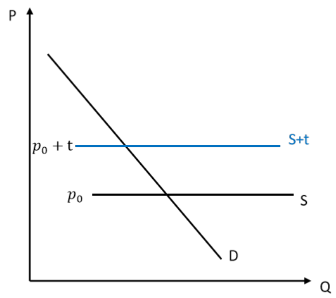


The demand price elasticity is 0. The tax burden is entirely borne by consumers

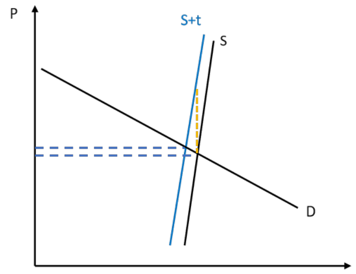


The demand price elasticity is infinite The tax burden is entirely borne by the producers

- Low price elasticity of supply $\Rightarrow$ Difficult to switch production $\Rightarrow$ Must sell even at lower prices $\Rightarrow$ Difficulty in shifting the tax $\Rightarrow$ Producers bear more tax burden.



Supply price elasticity is infinite, tax burden falls entirely on consumers.



Supply price elasticity is very small, tax burden falls almost entirely on producers.

6.1.3 Empirical Study on Tax Incidence: Doyle and Sampatharank (2008)

- **Topic**: Studies the tax incidence of gasoline sales tax.
- **Background**:
  - In the spring of 2000, gasoline prices in the Midwest surged, even exceeding $2 per gallon.

- It was an election year, and due to political pressure, state governments were inclined to reduce taxes.
- Two states suspended their 5% gasoline sales tax:
  - Indiana (IN): Suspended July 1, reimposed October 30
  - Illinois (IL): Suspended July 1, reimposed December 31

How to Estimate Tax Incidence

- **Gasoline Sales Tax Background**:
  - Besides sales tax, gasoline is also subject to an excise tax, i.e.,

$$\text{Retail Price} = \text{Price Received by Firms} + \text{Sales Tax} + \text{Excise Tax}$$

  - IL and IN allow all or part of the excise tax to be deducted from the sales tax base.
  - The sales tax base for gasoline in IL is approximately 90% of the retail price; in IN, it is approximately 80%.

- **Empirical Design**:
  - If the benefit of the tax cut is fully passed on to consumers, the retail gasoline price in IL should fall by 4.5% ($\frac{90\times5\%}{100}$).
  - If results show that after the tax cut, retail gasoline prices in IL fell by 2%, it implies that 44% (2/4.5) of the tax cut benefit is enjoyed by consumers.
  - Similarly, if results show that after reimposing the sales tax, retail gasoline prices in IL increased by 2%, it implies that 44% of the tax burden is borne by consumers.
  - Therefore, the empirical goal is to estimate the change in retail gasoline prices following the tax cut (and reimposition).

How to Estimate the Change in Retail Gasoline Prices

- **Single Difference Method**
  - Subtract the pre-tax cut price from the post-tax cut price in these two states.
  - $P_{IL,after} - P_{IL,before}$
  - Price changes are not necessarily due to the tax cut: changes in international oil prices, seasonal variations, etc.

- **Difference-in-Differences Method**
  - Compare with neighboring states.
  - Assumption: If there were no tax cuts, the trend of oil price changes in the two states would be the same

$$y_{it} = \beta_0 + \beta_1 Treat_i * Post_t + \beta_2 Treat_i + \beta_3 Post_t$$

- $Treat_i$: Whether it is the treatment group. $Treat_{IL} = 1$, $Treat_{IA} = 0$
- $Post_t$: Whether it is after the experiment time. $Post_{June} = 0$, $Post_{July} = 1$

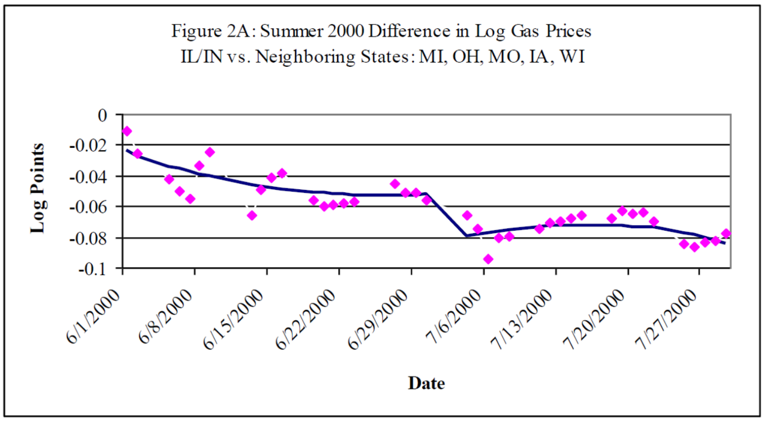
- $Treat_i * Post_t$: Whether state i at time t had a tax cut.

	June	July
IA	β_0	$\beta_0 + \beta_3$
IL	$\beta_0 + \beta_2$	$\beta_0 + \beta_1 + \beta_2 + \beta_3$
Difference	β_2	$\beta_1 + \beta_2$

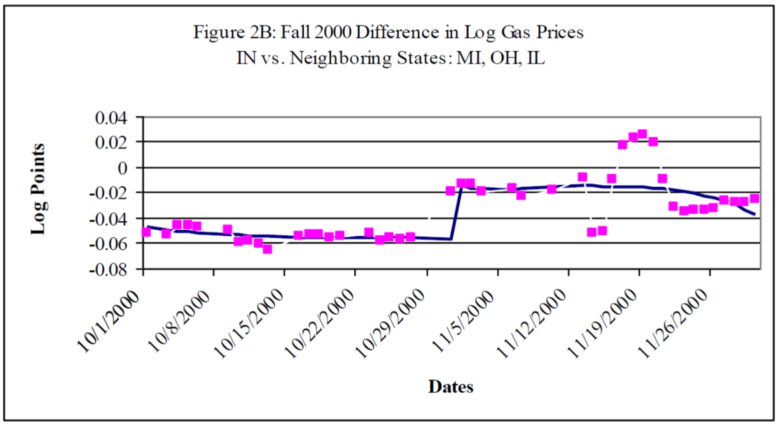
Empirical Strategy in this paper

$$\ln(\text{RetailPrice}_{sbt}) = \beta_0 + \beta_1 (IL \text{ or } IN)_s + \beta_2 \text{PostReform}_t + \beta_3 (IL \text{ or } IN)_s \times \text{PostReform}_t + \beta_4 \ln(\text{WholesalePrice})_s + \beta_5 X_5 + \delta_b + \varepsilon_{sbt}$$

- **First Regression**: Effect of the July 1 tax cut in IL and IN
  - (IL, IN) vs. (neighboring states)
  - (June) vs. (July)



- **Second Regression**: Effect of the October 30 tax reinstatement in IN
  - (IN) vs. (neighboring states)
  - (October) vs. (November)



- **Third Regression**: Effect of the December 31 tax reinstatement in IL
  - (IL) vs. (neighboring states)
  - (December) vs. (January)

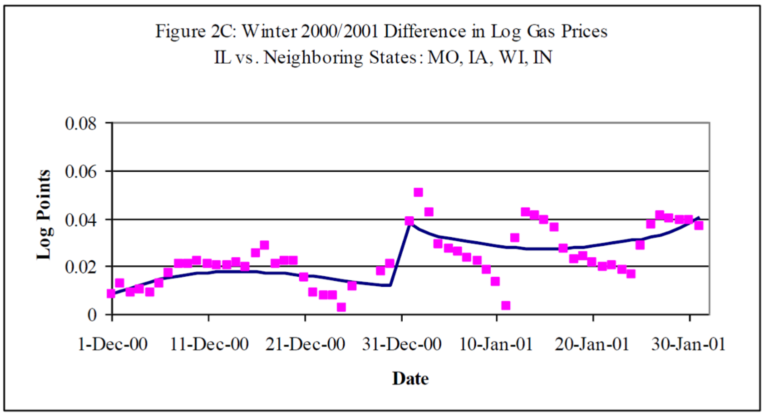


Figure 2C: Winter 2000/2001 Difference in Log Gas Prices
IL vs. Neighboring States: MO, IA, WI, IN

Table 2: Regression Results

A: July Tax Repeal

Dependent Variable:	Log(Retail Price)		
	(1)	(2)	(3)
Illinois or Indiana	-0.048	-0.013	-0.014
	(0.038)	(0.025)	(0.021)
Post July 1	-0.052	0.029	0.025
	(0.007)	(0.013)	(0.015)
(IL or IN)*Post July 1	-0.035	-0.029	-0.029
	(0.007)	(0.008)	(0.008)
Observations	29675	29675	29433
R-Squared	0.23	0.60	0.64
Mean of Dep. Var.	0.560	0.560	0.560
Controls:			
Wholesale Price	No	Yes	Yes
ZIP Codes Characteristics & Brand	No	No	Yes

Panel A: Prices observed June 27, June 28, July 5, July 6;
Standard errors are reported, clustered at the state level.

6.1: Regression Results: Tax Cut

About 70% of the tax cut benefit is enjoyed by consumers.

$$\frac{2.9}{4.5} = 64.4\%, \quad \frac{2.9}{4} = 72.5\%$$

- What identification method does this regression primarily use?
  - Difference-in-differences method.
- In Table 2, Panel A, what is the difference between the columns?
  - The control variables are different. From column (1) to column (3), wholesale price, ZIP codes characteristics, and Brand are sequentially added as control variables.

- Using the regression results in column (3) as an example, how much did the tax cut reduce retail gasoline prices?
  - 2.9%

Table II: Regression Results

B: October Tax Reinstatement		C: January Tax Reinstatement	
Dependent Variable:	Log(Retail Price)	Dependent Variable:	Log(Retail Price)
	(3)		(3)
Indiana	-0.053	Illinois	-0.005
	(0.007)		(0.021)
Post Oct. 31	-0.009	Post Jan. 1	-0.020
	(0.006)		(0.004)
IN*Post Oct. 31	$\boxed{0.040}$	IL*Post Jan. 1	$\boxed{0.037}$
	(0.006)		(0.004)
Observations	21884	Observations	7071
R-Squared	0.26	R-Squared	0.39
Mean of Dep. Var.	0.456	Mean of Dep. Var.	0.303

Models include full controls. Standard errors are reported, clustered at the state level.
Panel B: Prices observed Oct. 26, Oct. 27, Oct. 31, Nov. 1
Panel C: Prices observed Dec. 29, Dec. 30, Jan. 2, Jan. 3.

6.2: Regression Results: Tax Increase

Sales Tax Incidence by State

Indiana (IN): $\frac{4}{4} = 100\% \Rightarrow$ The entire tax burden falls on consumers.

Illinois (IL): $\frac{3.7}{4.5} = 82.2\% \Rightarrow 82.2\%$ of the tax burden falls on consumers.

Conclusion and Comments

- **Conclusion**
  - About 70% of the tax cut benefit was enjoyed by consumers.
  - 80%-100% of the tax increase burden was passed on to consumers.
- **Strengths of the Article**
  - Clear graphs, relatively clean regression.
  - Examined the effects of both tax cuts and tax increases.

6.1.4 Other Literature

- Hastings and Washington (2010)
  - Research topic: Tax (subsidy) incidence of food stamps.
  - Research background: Food stamps in Nevada are distributed in the first week of each month.
  - Research conclusion: During the week food stamps are distributed, food demand increases by 30%, and prices increase by 3%.
  - Part of the subsidy provided by food stamps is enjoyed by producers.
- Fabra and Reguant (2014)

  - About 86% of the increase in electricity generation costs due to carbon emissions trading is passed on to consumers through higher electricity prices.

- Suárez et al. (2016)
  - Tax incidence of state corporate tax: About 40% borne by owners, 30-35% borne by workers, and 25-30% borne by landowners.

- Hindriks and Serse (2019)
  - Tax incidence of excise tax on spirits: Over-shifted to consumers.

- Tax incidence of cigarette excise tax
  - Most literature finds that the tax burden is fully or even over-shifted to consumers.
  - Harding et al (2012: About 80%-90% is passed on to consumers.

- Zhou and Zhao (2020)
  - Research content: Who benefits from the 50% reduction in vehicle purchase tax?
  - Research conclusion: 85% is gained by consumers.

- Wang et al. (2022)
  - Tax incidence of China's value-added tax (general equilibrium method)
  - 39% passed on to labor, 61% passed on to capital, representative consumers bear none.

6.2 General Equilibrium

In the previous section, we learned about partial equilibrium analysis. However, there are several limitations of it which may bring challenges in research.

- The demand function represents the change in demand for a good in response to changes in its own price, holding other conditions constant.
  - However, taxes usually change not only the price of one good but also the prices of other goods.
    - For example, a value-added tax (VAT) affects a wide range of goods.
    - Partial equilibrium analysis is suitable for taxes levied on specific goods.
  - Taxes can also directly lead to changes in consumer income.
    - If coffee is a major industry in a country, taxing coffee will affect the income of various agents in the industrial chain.
  - Partial equilibrium analysis is suitable for goods with relatively small markets.

- Price changes caused by taxes can affect the demand for other goods.
  - Substitution effect: Other goods become relatively cheaper, increasing demand.
  - Income effect: For example, taxing food increases food prices, leading to higher food expenditure, reducing consumers' disposable income for other products, thus affecting the consumption of other goods. From this perspective, partial equilibrium analysis is suitable for goods with a small share of expenditure.

- It does not account for the distribution of tax incidence among factors of production.

6.2.1 General Equilibrium Analysis

- Considers multiple goods markets.
- Examines not only the incidence of taxes between producers and consumers but also their incidence among factors of production.
- Analyzes both consumption behavior and production behavior.

A Simple Example Considering Production: If a province taxes cola, explore its tax incidence.

- If cola demand is inelastic, consumers bear the tax burden.
- If cola demand is perfectly elastic, producers bear the tax burden.
  - Factors of production input: labor, capital, land, etc.
  - In the short run, labor is relatively elastic (can move to other provinces), while capital and land are relatively inelastic. The tax burden is likely borne by capital and land.
  - In the long run, capital can also move. The tax burden is likely borne only by land.

Classic General Equilibrium Model (Harberger JPE 1962)

- Two goods, two factor inputs (factors can move freely between sectors).
- Production functions:

$$X = F(K_X, L_X)$$
$$Y = G(K_Y, L_Y)$$

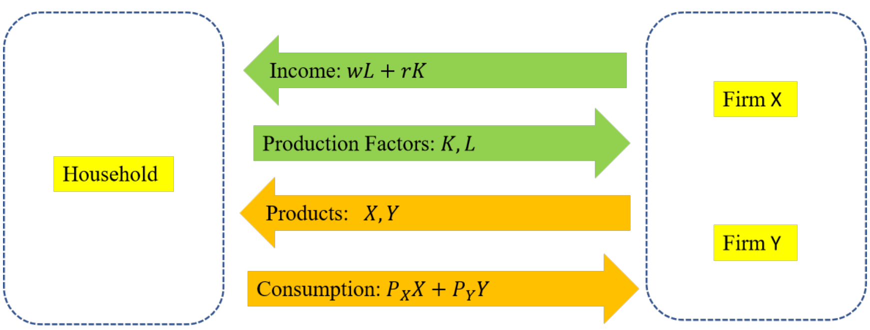
- One representative household with utility function:

$$U(X, Y)$$

- Factor endowment constraints:

$$K_X + K_Y = \bar{K}$$
$$L_X + L_Y = \bar{L}$$

- Note: Factors are owned by the representative household.



6.3: General Equilibrium Diagram

6.2.2 Taxing the Factor

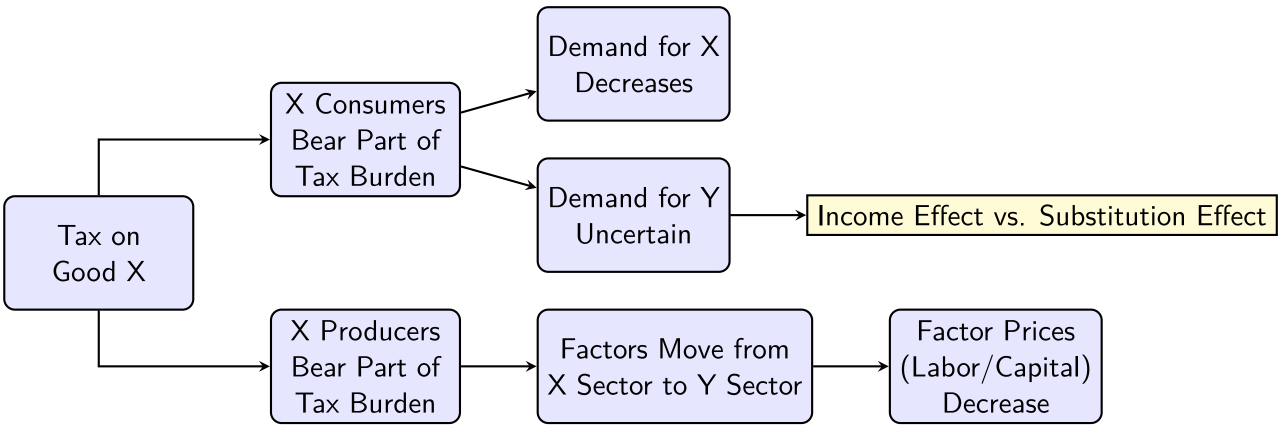
When it comes to taxation, there are different ways of taxing.

- Tax on capital
  - Tax on capital used only in producing X
  - Tax on capital used only in producing Y
  - Tax on all capital
- Tax on labor
  - Tax on labor used only in producing X
  - Tax on labor used only in producing Y
  - Tax on all labor
- Tax on goods
  - Tax only on X
  - Tax only on Y
  - Tax on all goods

Taking Tax Only on X as an Example

- **Consumption side**: Tax on X, price of X (including tax) increases.
  - Substitution effect: Demand for X falls, demand for Y increases.
  - Income effect: Demand for X falls, demand for Y falls.
  - Demand for X falls, demand for Y is uncertain.
  - If X and Y are substitutes, demand for Y increases. This will cause the price of Y to rise, so part of the tax burden is borne by consumers of Y.
- **Production side**
  - Production of X will decrease, factors of production will move from the X sector to the Y sector.
  - Assume X is labor-intensive (e.g., food) and Y is capital-intensive (e.g., manufactured goods).
  - A large amount of labor flows out of the X sector and into the Y sector. But the Y sector needs capital more.
  - Therefore, the price of labor (wage) falls, and the price of capital (rental rate) rises relatively.
  - Workers lose, capital owners gain relatively.

Assume X is labor-intensive (e.g., food) and Y is capital-intensive (e.g., manufactured goods).



6.4: Tax Incidence and Factor Mobility

6.2.3 Solving for Equilibrium

- **Consumer Utility Maximization**
  - Given $\{P_X, P_Y, w, r\}$, the consumer maximizes utility.
  - Derive the demand for goods $\{X_D, Y_D\}$ given prices.
- **Producer Profit Maximization**
  - Given $\{P_X, P_Y, w, r\}$, the producer maximizes profit.
  - Derive the supply of goods $\{X_S, Y_S\}$ and the demand for factors $\{K_X, L_X, K_Y, L_Y\}$ given prices.
- **Market Equilibrium**
  - Demand for goods = Supply of goods, to find goods prices.
  - Demand for factors = Supply of factors, to find factor prices.

6.2.3.1 Utility Maximization

- Utility function: $U(X, Y) = X^\alpha Y^\beta$
- Utility maximization:

$$\max X^\alpha Y^\beta$$
$$\text{s.t. } P_X X + P_Y Y = I$$

- Solution:

$$X = \frac{\alpha}{\alpha+\beta}\frac{I}{P_X}, \quad Y = \frac{\beta}{\alpha+\beta}\frac{I}{P_Y}$$

- Conclusions:
  - The expenditure share of good X is $\frac{\alpha}{\alpha+\beta}$, and for good Y it is $\frac{\beta}{\alpha+\beta}$.
  - The cross-price elasticity between the two goods is zero.

6.2.3.2 Firm Profit Maximization

- Given product and factor prices $\{P, w, r\}$, choose optimal labor and capital quantities $\{K, L\}$ to maximize profit.

$$\max PF(K, L) - wL - rK$$

First-order conditions:

$$(K) : PF_K(K, L) = r$$
$$(L) : PF_L(K, L) = w$$

Therefore:

$$\text{Real rental rate} = \text{Marginal product of capital: } \frac{r}{P} = F_K(K, L) \equiv MPK$$

$$\text{Real wage} = \text{Marginal product of labor: } \frac{w}{P} = F_L(K, L) \equiv MPL$$

6.2.3.3 Firm Profit Maximization

- **Factor Payments**
  - Total "real wage bill" paid to all workers: $MPL \times L$
  - Total "real capital income" paid to all capital owners: $MPK \times K$
- **Think about this:**
  - Besides payments to labor and capital owners, do firms have any additional profit?
  - In a perfectly competitive market, if the production function exhibits constant returns to scale, revenue is only used to pay labor and capital owners.

Definition 6.2.1: Constant Returns to Scale (CRS) production function

If all factors increase by α%, output also increases by α%.

$$F(\lambda K, \lambda L) = \lambda F(K, L)$$

- Differentiate both sides with respect to λ:

$$F_K(\lambda K, \lambda L)K + F_L(\lambda K, \lambda L)L = F(K, L)$$

- Set $\lambda = 1$, then $F_K(K, L)K + F_L(K, L)L = F(K, L)$
- That is: $F(K, L) = MPK \times K + MPL \times L$
- **Why do firms have profits in reality?**
  - In reality, most firms own rather than rent their capital.
  - Economic profit $= Y - MPL \times L - MPK \times K$ (equals 0 under CRS).

- Accounting profit = Economic profit + $MPK \times K$ (not zero).

- Economist Paul Douglas noticed that the division of national income between capital and labor remained roughly constant over long periods.
- He asked mathematician Charles Cobb: If factor payments equal their marginal products, what production function could yield constant factor shares?
- If the production function satisfies $F(K, L) = AK^{\alpha}L^{1-\alpha}$, then:
  - It exhibits constant returns to scale.
  - $MPK \times K = \alpha Y$, $MPL \times L = (1-\alpha)Y$.

6.2.3.4 Production Function Setup

- Production function: $X = AK^{\alpha}L^{1-\alpha}$
- Firm profit maximization problem:

$$\max PAK^{\alpha}L^{1-\alpha} - rK - wL$$

- First-order conditions:

$$\begin{aligned}(K) : P\alpha AK^{\alpha-1}L^{1-\alpha} &= r \Rightarrow \alpha PX = rK \\ (L) : P(1-\alpha)AK^{\alpha}L^{-\alpha} &= w \Rightarrow (1-\alpha)PX = wL\end{aligned}$$

- Conclusions:
  - Factor prices = Product price × Marginal product of the factor.
  - Labor's income share is α, capital's income share is $(1-\alpha)$.

6.2.4 Specific Functional Forms of General Equilibrium

- Two goods with the following production functions:

$$X = K_X^{\frac{1}{6}}L_X^{\frac{5}{6}}, \quad Y = K_Y^{\frac{5}{6}}L_Y^{\frac{1}{6}}$$

- One representative household
  - Endowment: Owns 1 unit of labor and 1 unit of capital.
  - Utility function: $U(X, Y) = XY$
- Perfectly competitive markets, no taxes initially. Factors can move freely.
- **Find:**
  - Equilibrium prices: P_X, P_Y, w, r
  - Equilibrium quantities and factor inputs: X, Y, K_X, K_Y, L_X, L_Y

Guess the Equilibrium State

- How do the outputs of the two sectors compare? X vs Y
- How do the prices of the two goods compare? P_X vs P_Y
- Which factor price is higher? w vs r
- How is capital allocated between the two sectors? K_X vs K_Y
- How is labor allocated between the two sectors? L_X vs L_Y

Consumer Utility Maximization

$$X_D = \frac{I}{2P_X}, \quad Y_D = \frac{I}{2P_Y} \quad \text{(where } I = w + r\text{)}$$

Producer Profit Maximization

$$\text{X profit max: } \frac{5}{6}P_X X_S = wL_X, \quad \frac{1}{6}P_X X_S = rK_X$$
$$\text{Y profit max: } \frac{1}{6}P_Y Y_S = wL_Y, \quad \frac{5}{6}P_Y Y_S = rK_Y$$

(Think: Which is labor-intensive, X or Y?)

Market Equilibrium

$$K_X + K_Y = 1, \quad L_X + L_Y = 1$$
$$X_D = X_S, \quad Y_D = Y_S$$

Solving for factor allocation and output $(X, Y, K_X, K_Y, L_X, L_Y)$

- From utility maximization: $P_X X = P_Y Y$.
- Combining with profit maximization conditions for X and Y:

$$\frac{5P_X X}{P_Y Y} = \frac{L_X}{L_Y} = 5, \quad \frac{P_X X}{5P_Y Y} = \frac{K_X}{K_Y} = \frac{1}{5}$$

- Combining with market equilibrium:

$$L_X = \frac{5}{6}, \quad L_Y = \frac{1}{6}, \quad K_X = \frac{1}{6}, \quad K_Y = \frac{5}{6}$$
$$X = \left(\frac{1}{6}\right)^{\frac{1}{6}} \left(\frac{5}{6}\right)^{\frac{5}{6}}, \quad Y = \left(\frac{5}{6}\right)^{\frac{5}{6}} \left(\frac{1}{6}\right)^{\frac{1}{6}}$$

Solving for Commodity and Factor Prices

- With four goods, there are only three relative prices. We can normalize by setting the price of labor $w = 1$.
- From X's profit maximization:

$$5 = \frac{wL_X}{rK_X} \quad \Rightarrow \quad r = w = 1$$

$$\frac{5}{6}P_X X = wL_X \quad \Rightarrow \quad P_X = \left(\frac{1}{6}\right)^{-\frac{1}{6}} \left(\frac{5}{6}\right)^{-\frac{5}{6}}$$

- From Y's profit maximization:

$$\frac{1}{6}P_Y Y_S = wL_Y \quad \Rightarrow \quad P_Y = \left(\frac{1}{6}\right)^{-\frac{1}{6}}\left(\frac{5}{6}\right)^{-\frac{5}{6}}$$

Think about this

1. Does $I = w + r$ hold?

$$I = 2P_X X = 2\left(\frac{1}{6}\right)^{\frac{1}{6}}\left(\frac{5}{6}\right)^{\frac{5}{6}}\left(\frac{1}{6}\right)^{-\frac{1}{6}}\left(\frac{5}{6}\right)^{-\frac{5}{6}} = 2 = w + r$$

2. Why wasn't $I = w + r$ used in the solution process?
   - If $I = w + r$ were used, we would only need three of the four market equilibrium conditions; the fourth market would automatically clear.
   - In the actual solution process, four equilibrium conditions were used, so $I = w + r$ holds automatically.
   - The implication of $I = w + r$: Total expenditure = Total income.

Equilibrium State

- How do the outputs of the two sectors compare? $X = Y$
- How do the prices of the two goods compare? $P_X = P_Y$
- Which factor price is higher? $w = r$
- How is capital allocated between the two sectors? $K_X < K_Y$
- How is labor allocated between the two sectors? $L_X > L_Y$

6.2.5 Tax Incidence

- Assume a 50% ad valorem tax is levied on Y (exclusive tax).
- **Assumption about fiscal spending**:
  - Can assume all tax revenue is transferred back to the representative household.
  - Can also assume the government has the same utility function as the household. In this case, whether it's the government or the private sector, half of the income is spent on X and half on Y. Therefore, the demand functions for the whole society remain $X_D = \frac{I}{2P_X}$ and $Y_D = \frac{I}{2P_Y}$.
- **Exploring tax incidence**:
  - After the tax, how do prices change: P_X, P_Y, w, r?
  - After the tax, how do the utilities of workers and capital owners change?
    * Assuming there are two representative households, one owning 1 unit of labor, the other owning 1 unit of capital.

* Under the utility function $U = XY$, the demand functions won't change; half of total societal income is still spent on X and half on Y.
* But we can calculate the utility of the worker and the capital owner separately.

Assumptions

- A 50% ad valorem tax is levied on Y.
- Let P_Y be the price received by the firm, then the price paid by consumers is $\frac{3}{2}P_Y$.
- All tax revenue is transferred to the representative household.
- **Consumer Utility Maximization (different from pre-tax)**

$$X_D = \frac{I}{2P_X}, \quad Y_D = \frac{I}{3P_Y} \quad (\text{where } I = w + r + \frac{1}{2}P_Y Y)$$

- **Producer Profit Maximization (same as pre-tax)**

$$\text{X profit max: } \frac{5}{6}P_X X_S = wL_X, \quad \frac{1}{6}P_X X_S = rK_X$$
$$\text{Y profit max: } \frac{1}{6}P_Y Y_S = wL_Y, \quad \frac{5}{6}P_Y Y_S = rK_Y$$

- **Market Equilibrium (same as pre-tax)**

$$K_X + K_Y = 1, \quad L_X + L_Y = 1$$
$$X_D = X_S, \quad Y_D = Y_S$$

Solving for factor allocation and output $(X, Y, K_X, K_Y, L_X, L_Y)$

- From utility maximization: $2P_X X = 3P_Y Y$.
- From profit maximization conditions for X and Y:

$$\frac{5P_X X}{P_Y Y} = \frac{L_X}{L_Y}, \quad \frac{P_X X}{5P_Y Y} = \frac{K_X}{K_Y}$$

- Combining with market equilibrium:

$$L_X = \frac{15}{17}, \quad L_Y = \frac{2}{17}, \quad K_X = \frac{3}{13}, \quad K_Y = \frac{10}{13}$$
$$X = \left(\frac{3}{13}\right)^{\frac{1}{6}} \left(\frac{15}{17}\right)^{\frac{5}{6}}, \quad Y = \left(\frac{10}{13}\right)^{\frac{1}{6}} \left(\frac{2}{17}\right)^{\frac{1}{6}}$$

Solving for goods and factor prices

- With four goods, there are only three relative prices. We can normalize by setting the price of labor, i.e., let $w = 1$.
- From the profit maximization problem of X:

$$5 = \frac{wL_X}{rK_X} \implies r = \frac{13}{17}$$

- From the profit maximization problem of X:

$$\frac{5}{6}P_X X = wL_X \implies P_X = \frac{6}{5}\left(\frac{65}{17}\right)^{\frac{1}{6}}$$

- From the profit maximization problem of Y:

$$\frac{1}{6}P_Y Y_S = wL_Y \implies P_Y = 6\left(\frac{13}{85}\right)^{\frac{5}{6}}$$

- **Check if $I = w + r + \frac{1}{2}P_Y Y$ holds.**

$$w + r + \frac{1}{2}P_Y Y = 1 + \frac{13}{17} + \frac{1}{2} \times \frac{12}{17} = \frac{36}{17}$$
$$X_D = \frac{I}{2P_X} \Rightarrow I = 2P_X X = 2 \times \frac{6}{5}wL_X = \frac{36}{17}$$

- **If the tax revenue had not been transferred to the representative household, which part of the solution process would change?**
  - The income I in the consumer's utility maximization would not include the transfer $\frac{1}{2}P_Y Y$. The demand functions would still be $X_D = I/(2P_X)$ and $Y_D = I/(2P_Y)$ but with $I = w + r$. This would change the equilibrium conditions and the final solution.

6.2.5.1 Exploring Tax Incidence: Price Changes

	Pre-tax	Post-tax	Comparison	Intuitive Explanation
w	1	1	Cannot explain that labor price remains unchanged before and after taxation.	
r	1	$\frac{13}{17}$	After taxation, capital becomes cheaper (relative to labor).	Tax is imposed on Y(capital-intensive). Factors of production flow from Y to X (labor-intensive). Since producing X does not require as much capital, capital becomes relatively cheaper.
P_X	$\left(\frac{1}{6}\right)^{-\frac{1}{6}}\left(\frac{5}{6}\right)^{-\frac{5}{6}} \approx 1.57$	$\frac{6}{5}\left(\frac{65}{17}\right)^{\frac{1}{6}} \approx 1.50$	After taxation, X becomes cheaper.	Since the cross-price elasticity of demand for X with respect to Y is zero, the price change of Y due to taxation does not affect the demand for X. However, more factors flow into the X sector, increasing the supply of X, leading to a decrease in its price.
P_Y	$\left(\frac{1}{6}\right)^{-\frac{1}{6}}\left(\frac{5}{6}\right)^{-\frac{5}{6}} \approx 1.57$	$6\left(\frac{13}{85}\right)^{\frac{5}{6}} \approx 1.25$	$1.25 < 1.57$: the price received by Y producers decreases. $1.25 \times 1.5 = 1.857 > 1.57$, the price paid by consumers for Y increases	The tax burden is shared by both producers and consumers. Therefore, the price paid by consumers increases, while the price received by producers decreases.

6.5: Comparison Before and After Taxation

6.2.5.2 Exploring Tax Incidence: Utility Change

$$X_D = \frac{I}{2P_X^c}, \quad Y_D = \frac{I}{2P_Y^c}, \quad U = \frac{I^2}{4P_X^c P_Y^c}$$

	Pre-tax	Post-tax
P_X^c	1.57	1.50
P_Y^c	1.57	1.88
I	$w + r = 2$	$w + r + \frac{1}{2}P_Y^c Y = 2.12$
U	0.41	0.40

Utility decreases after tax. Why?
First Fundamental Theorem of Welfare Economics!

6.2.5.3 Exploring Tax Incidence: Utility of Laborer and Capital Owner

Assume one representative household owns 1 unit of labor, and another representative household owns 1 unit of capital. Tax revenue is transferred equally to both households.

- **Laborer:**
  - Income change: Labor income is still 1, but there is a government transfer, so income (in terms of labor numeraire) increases.
  - Goods price change: X is cheaper, Y is more expensive.
  - Since the price change in X is small and in Y is large, the overall price increases.
  - Utility change is uncertain.
- **Capital Owner:**
  - Income change: After tax, capital income decreases, but there is a government transfer. (Intuitively, part of the government tax revenue is also transferred to the worker, so the increase in transfer is less than the decrease in capital income.)
  - Goods price change: X is cheaper, Y is more expensive, overall price level increases.
  - The utility of the capital owner likely decreases.

	Pre-tax	Post-tax
Prices		
P_X	1.57	1.50
P_Y	1.57	1.88
Income		
Labor	$I = 1$	$I = 1 + \frac{1}{2}\left(\frac{1}{2}P_Y Y\right) = 1.18$
Capital	$I = 1$	$I = 1 + \frac{1}{2}\left(\frac{1}{2}P_Y Y\right) = 0.94$
Utility		
Labor	0.101	0.123
Capital	0.101	0.078
Sum of Utilities	0.202	0.201

Laborer utility increases, capital owner utility decreases. Total utility decreases.

Chapter 7 Optimal Tax

How should a government design taxes to raise revenue while minimizing economic inefficiency? This chapter explores the core principles of optimal taxation, beginning with the measurement of efficiency losses caused by taxes. We introduce the concept of *deadweight loss* as a key indicator of inefficiency, and discuss how *equivalent variation* and *compensating variation* can be used to quantify the excess burden of taxation.

The central question we address is: How should taxes be structured to minimize efficiency loss? Building on the foundational work of Ramsey (1927), we examine tax rules that minimize deadweight loss, using the equivalent variation approach to measure the excess burden. This framework provides the theoretical basis for understanding trade-offs in tax policy and the design of efficient tax systems.

7.1 Excess Burden of Taxation

Review from *Principle of Public Finance*,

- Definition of excess burden: The welfare loss imposed on taxpayers beyond the tax revenue collected by the government.
- A tax generates no excess burden if it does not create substitution effects; such a tax is said to be neutral.
- Lump-sum tax (head tax) is neutral because it does not distort relative prices or behavior.
  - However, lump-sum taxes are generally considered unfair due to their regressive nature.

How to Measure Excess Burden?There are two main approaches to measuring excess burden:

- Social welfare approach: Using Marshallian consumer and producer surplus.
- Equivalent variation and compensating variation: Hicksian measures of welfare change.

7.1.1 Social Welfare Approach to Measuring Excess Burden

7.1.1.1 Model Setup

- Two goods:
  - x: taxed good

  - y: untaxed good (numeraire)

- Consumer: quasi-linear utility $u(x) + y$
- Producer: cost of producing x is $c(x)$
- Prices:
  - y is the numeraire (price = 1)
  - Consumer price of x: p
  - Tax per unit of x: t
- Other notation:
  - Consumer income: Z
  - Tax revenue: $R = t \cdot x$
  - η_S, η_D: supply elasticity and demand elasticity (absolute values) of good x

$$\eta_S = \frac{\partial x_S}{\partial P} \cdot \frac{P}{x_S}, \quad \eta_D = -\frac{\partial x_D}{\partial P} \cdot \frac{P}{x_D}$$

Deriving the Demand Curve
Consumer maximizes utility subject to budget constraint:

$$\max_{x,y} \; u(x) + y \quad \text{s.t.} \quad px + y = Z$$

Substituting the constraint into the objective function:

$$\max_{x} \; u(x) + Z - px$$

First-order condition:

$$p = u'(x)$$

The demand curve for good x is given by the marginal utility function:

$$\boxed{p = u'(x)}$$

Deriving the Supply Curve
Producer maximizes profit:

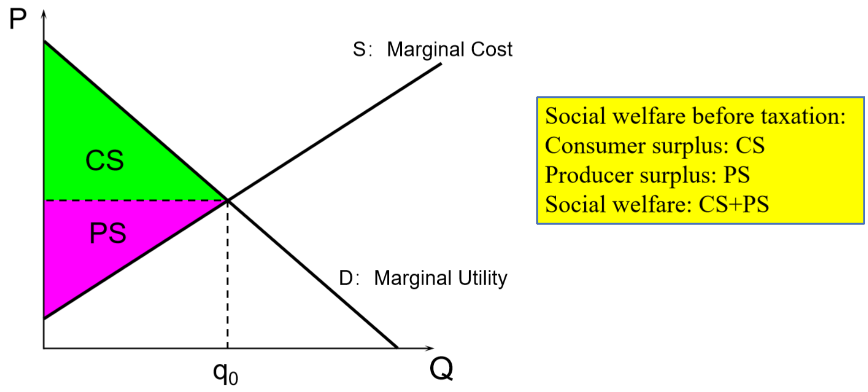
$$\max_{x} \; px - c(x)$$

First-order condition:

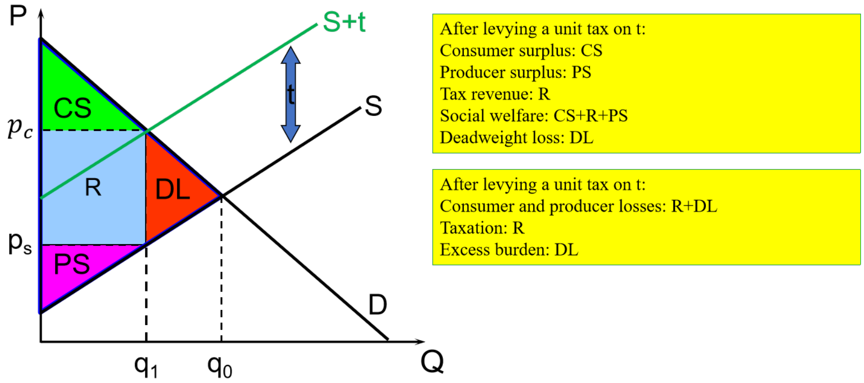
$$p = c'(x)$$

The supply curve for good x is given by the marginal cost function:

$$\boxed{p = c'(x)}$$



7.1: Market Equilibrium



7.2: Market Equilibrium after Taxation - Deadweight Loss

How to Measure Deadweight Loss

Starting from an initial state with no tax, introduce a small tax $d\tau$:

$$DL = \frac{1}{2}|dQ|\,d\tau$$

Using the definition of demand elasticity $\eta_D = -\frac{\partial D}{\partial P}\frac{P}{Q}$:

$$|dQ| = \eta_D\, dP \frac{Q}{P}$$

From partial equilibrium analysis of tax incidence:

$$dP = \frac{\eta_S}{\eta_S + \eta_D}\, d\tau$$

Substituting:

$$\boxed{DL = \frac{1}{2}\frac{\eta_D \eta_S}{\eta_S + \eta_D} Q\,(d\tau)^2}$$

- Deadweight loss increases with the square of the tax rate $(d\tau)^2$.
  - Implication: Spread taxes broadly across many goods to keep individual tax rates low.

- Deadweight loss is increasing in both demand elasticity η_D and supply elasticity η_S.

$$DL = \frac{1}{2}\frac{1}{\frac{1}{\eta_S}+\frac{1}{\eta_D}}\frac{Q}{P}(dt)^2$$

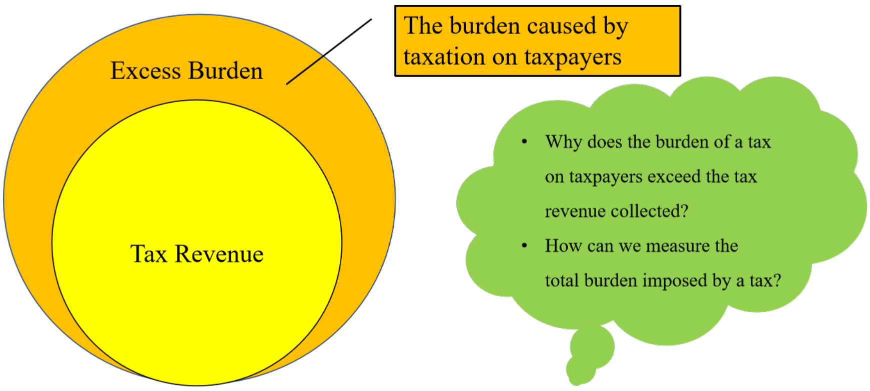
- Policy implication: Tax goods with lower elasticities more heavily (e.g., food, medicine) to minimize deadweight loss.
- Problem: This approach is unfair (regressive), as lower-income households spend a larger share of their income on necessities.

Now, we measure Excess Burden through Equivalent Variation and Compensating Variation. Before that, we give the definition and the real-world example about excess burden.

Definition 7.1.1: Excess Burden

Excess burden is the welfare loss imposed on taxpayers beyond the tax revenue collected by the government.

$$\text{Excess Burden} = \text{Total burden on taxpayers} - \text{Tax revenue}$$



7.3: Visual Representation of Excess Burden

Before diving into the algebra of excess burden, let us consider two real-world stories that vividly illustrate how taxes can distort behavior and create inefficiency. These historical examples—one from England, one from the Netherlands—show that the deadweight loss of taxation is not just a theoretical concept, but a force that shaped architecture, public health, and everyday life for centuries.

The Window Tax of King William III (1696)

In 1696, King William III introduced a window tax in England to raise revenue. The tax was based on the number of windows in a dwelling—the more windows, the higher the tax. To avoid the levy, many homeowners bricked up their windows, leading to dark, poorly ventilated homes. This behavioral response created a classic deadweight loss: people altered their housing choices not because they preferred fewer windows, but purely to avoid taxation. The tax distorted behavior, reduced welfare, and was eventually repealed in 1851 due to public health concerns. It remains a powerful historical example of how taxes can create unintended efficiency losses.

Window tax in England (1696-1851)

The Dutch Door Width Story

In 17th-century Amsterdam, the government taxed houses based on door size—wider doors meant higher taxes. To minimize their tax burden, residents built extremely narrow doors while making windows much larger. This creative avoidance created a lasting architectural legacy: even today, Amsterdam's canal houses feature disproportionately small doors, and you can still see hooks on gables used to hoist furniture through windows . Like England's window tax, this policy generated deadweight loss—people endured inconvenience (narrow entrances, complicated moves) while the government collected little revenue, as everyone simply adapted by shrinking their doors.

Tax on the width of house in Holland

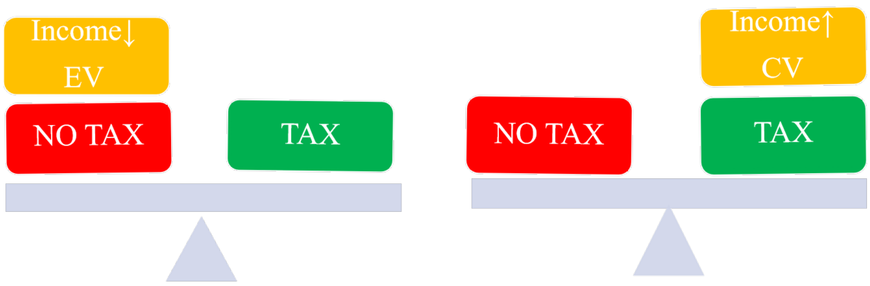
7.1.2 Equivalent Variation (EV) and Compensating Variation (CV)

7.1.2.1 Answer for Pervious Questions

- Why does the burden of a tax on taxpayers exceed the tax revenue collected?
  - Taxes distort relative prices, leading to substitution effects that reduce economic efficiency.
  - In addition to transferring purchasing power to the government (income effect), taxes cause consumers and producers to change their behavior (substitution

effect), creating an excess burden (deadweight loss).

- How can we measure the total burden imposed by a tax?
  - Equivalent Variation (EV): The amount of money that would need to be taken from the consumer (at pre-tax prices) to give the same utility loss as the tax.
  - Compensating Variation (CV): The amount of money that would need to be given to the consumer (after the tax is imposed) to restore their original utility level.



7.4: Equivalent Variation and Compensating Variation

Summary

- The total burden of a tax consists of:
  - Tax revenue: Transfer from consumers to government
  - Excess burden: Deadweight loss from distorted choices
- EV and CV are two money-metric measures of the total welfare loss from taxation.
- The difference between the total burden and tax revenue is the excess burden.

7.1.2.2 Measuring Excess Burden: Model Setup

- Two goods: X and Y
- A specific tax t is imposed on each unit of X
- Assume perfectly elastic supply (supply elasticity is infinite), so the entire tax burden falls on consumers

Which Tax Would Consumers Prefer?

Given the excess burden concept, in the previous example, which tax method would consumers prefer?

- Option A: A specific tax t per unit of X
- Option B: A lump-sum tax of $t \cdot Q_x^B$ (the same revenue collected directly)

7.1: Before and After Tax

Before Tax	
Prices	(P_x, P_y)
Consumption Bundle	Point A: (Q_x^A, Q_y^A)
After Tax	
Prices	$(P_x + t, P_y)$
Consumption Bundle	Point B: (Q_x^B, Q_y^B)
Tax Revenue	$t \cdot Q_x^B$

Review from Public Finance

- A tax generates no excess burden if it does not create substitution effects; such a tax is said to be neutral.
- Lump-sum tax is neutral because it does not distort relative prices or behavior.
- A specific tax is equivalent to a lump-sum tax of EV (Equivalent Variation).
- Definition of excess burden: $EV > t \cdot Q_x^B$
- Therefore, a specific tax is worse than a lump-sum tax with revenue $t \cdot Q_x^B$.

e.g. 11: Illustrative Example: Coca-Cola vs. Pepsi

7.2: Consumption Choices Before and After Tax

	Before Tax		**After Tax**	
	Price	Quantity	Price	Quantity
Coca-Cola	3	10	6	1
Pepsi	3	10	3	18

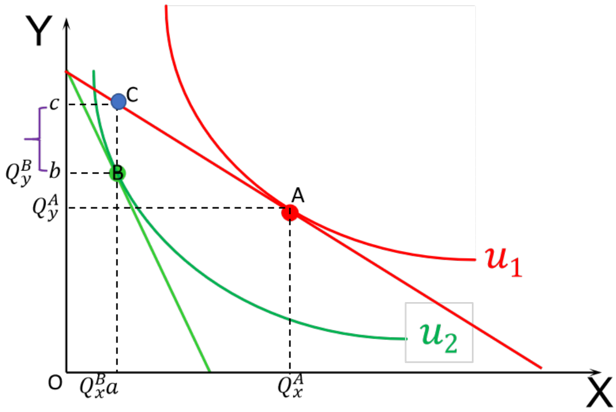
- Tax revenue: $t \times Q_x^B = 3 \times 1 = 3$
- How do we measure the burden imposed by this tax?

7.1.2.3 Measuring Excess Burden: Graphical Solution

- Budget constraint at point B:

$$Oa \cdot P_x + Ob \cdot P_y + T = I$$

The consumer can purchase Oa of X and Ob of Y.



7.5: Tax Revenue

- Budget constraint at point C:

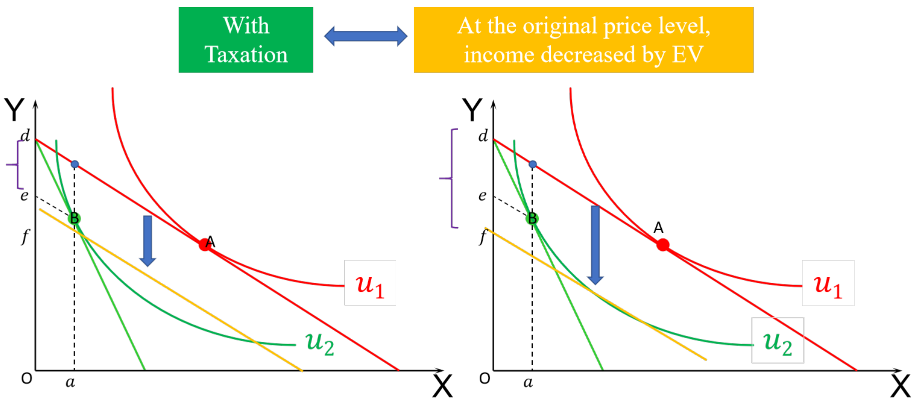
$$Oa \cdot P_x + Oc \cdot P_y = I$$

The consumer can purchase Oa of X and Oc of Y.

- Tax revenue:

$$T = bc \cdot P_y$$

where $bc = Ob - Oc$ represents the reduction in Y consumption due to the tax.



7.6: Equivalent Variation Measures Excess Burden: EV

- At point d:

$$Od \cdot P_y = I$$

The consumer can purchase Od units of Y with full income I at pre-tax prices.

- At point f:

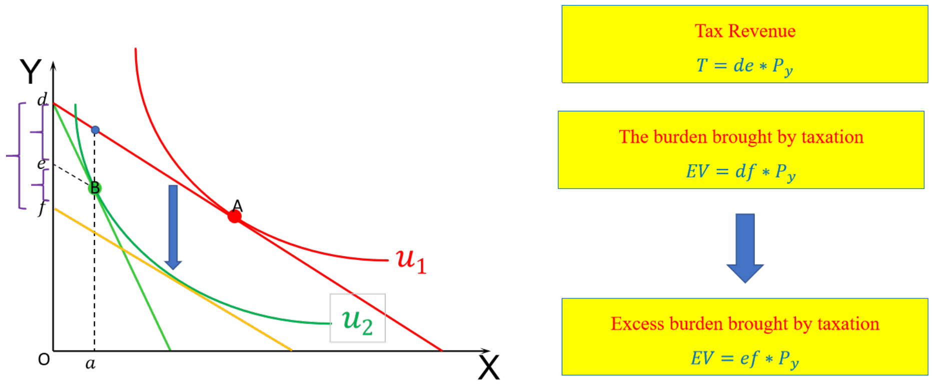
$$Of \cdot P_y = I - EV$$

The consumer can purchase Of units of Y after having EV (equivalent variation) taken away at pre-tax prices.

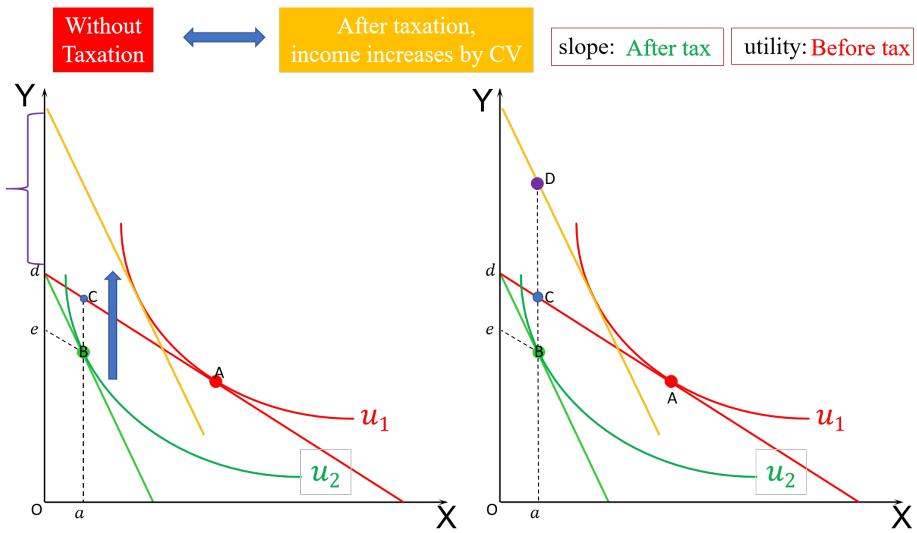
- The burden imposed by the tax (Equivalent Variation):

$$EV = df \cdot P_y$$

where $df = Od - Of$ represents the reduction in Y consumption equivalent to the welfare loss from the tax.



7.7: Equivalent Variation Measures Excess Burden: Excess Burden



7.8: Compensation Variation Measures Excess Burden: CV

- At point B:

$$Oa \cdot (P_x + t) + aB \cdot P_y = I$$

The consumer can purchase Oa units of X and aB units of Y at post-tax prices.

- At point D:

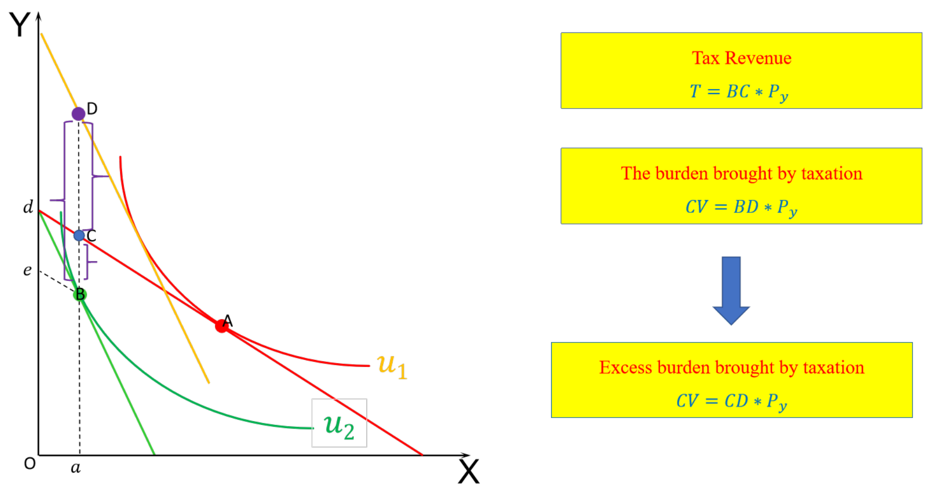
$$Oa \cdot (P_x + t) + aD \cdot P_y = I + CV$$

The consumer can purchase Oa units of X and aD units of Y at post-tax prices after receiving CV (compensating variation) to restore original utility.

- The burden brought by taxation (Compensating Variation):

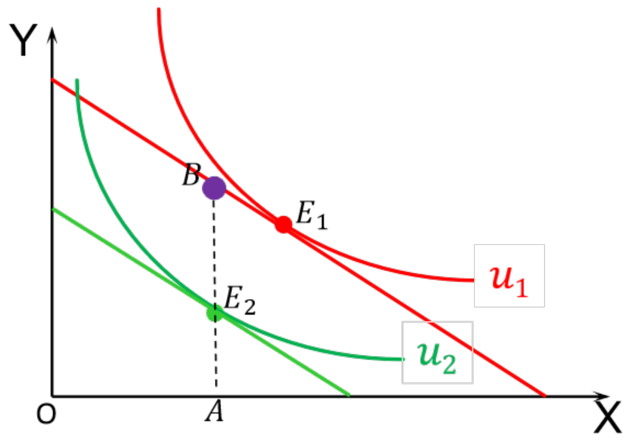
$$CV = BD \cdot P_y$$

where $BD = aD - aB$ represents the additional Y consumption needed to compensate the consumer for the tax at post-tax prices.



7.9: Compensation Variation Measures Excess Burden: Excess Burden

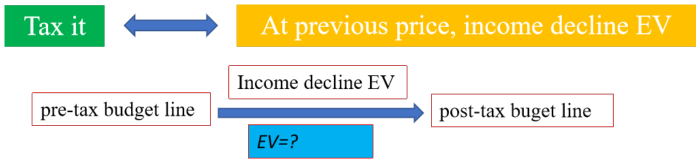
Case: What if the relative price remains unchanged



7.10: Constant Relative Price

- Assumption: Relative prices remain constant before and after taxation.
- Pre-tax prices: P_x, P_y
- Post-tax prices: $P_x + t_x, P_y + t_y$
- Constant relative price condition:

$$\frac{P_x}{P_y} = \frac{P_x + t_x}{P_y + t_y}$$



At original prices, income decreases by EV:

$$P_x x + P_y y = I - EV$$

After the income reduction, the budget line passes through point E_2:

$$P_x \cdot OA + P_y \cdot AE_2 = I - EV$$

From the post-tax budget constraint at point E_2:

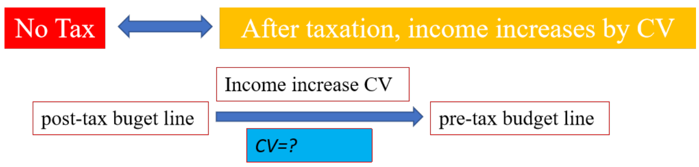
$$P_x \cdot OA + P_y \cdot AE_2 + T = I$$

Therefore:

$$\boxed{EV = T}$$

Note

When relative prices remain constant, the excess burden measured by equivalent variation is zero.



After the tax is imposed, income increases by CV at post-tax prices:

$$(P_x + t_x)x + (P_y + t_y)y = I + CV$$

After this income increase, the budget line passes through point B:

$$(P_x + t_x) \cdot OA + (P_y + t_y) \cdot AB = I + CV$$

From the pre-tax budget constraint at point E_1:

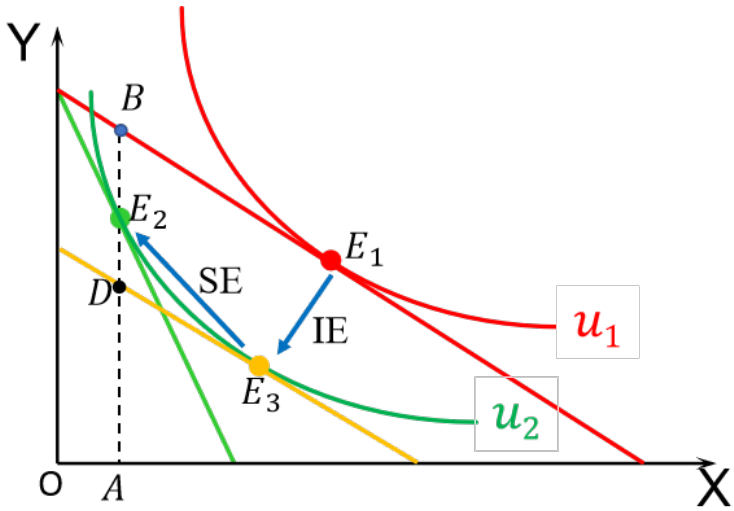
$$P_x \cdot OA + P_y \cdot AB = I$$

Compensating variation can be expressed as:

$$CV = t_x \cdot OA + t_y \cdot AB = T + t_y \cdot BE_2$$

Note

When relative prices remain constant, the excess burden measured by compensating variation is $t_y \cdot BE_2$.



7.11: Excess Burden and Substitution Effect

When There Is No Substitution Effect

- If there is no substitution effect:
  - Points E_2 and E_3 coincide (compensated and final bundles are the same).
  - Points D and E_2 coincide.
  - Excess burden is zero — the tax generates only an income effect, no distortion.
- This occurs when relative prices remain unchanged (uniform taxation) or when demand is perfectly inelastic.
- Excess burden is caused by *Substitution Effect.*

- The excess burden approach may overestimate the welfare loss from taxation because:
  - It only considers the tax revenue side, ignoring:
  - The redistributive effects of taxation (equity considerations)
  - The benefits of public goods financed by the tax revenue
  - A complete welfare analysis must weigh the deadweight loss against the social value of government spending.
- Relationship Between the Three Measures
  - The connections between Equivalent Variation (EV), Compensating Variation (CV), and Marshallian Consumer Surplus are beyond the scope of this course.
  - Interested students may explore the literature on welfare measurement (e.g.,

7.1.3 Another Perspective on Excess Burden: Review of Welfare Theorems

Individual Rationality

- Consumer utility maximization:

$$MRS_{XY} = \boxed{\frac{P_X}{P_Y}}$$

- Producer profit maximization:

$$\boxed{\frac{P_X}{P_Y}} = MRT_{XY}$$

Collective Rationality

- Pareto optimality requires:

$$MRS_{XY} = MRT_{XY}$$

Welfare Implications of Taxing Good X

When a tax t is imposed on good X:

- Consumer maximizes utility:

$$MRS_{XY} = \boxed{\frac{P_X(1+t)}{P_Y}}$$

- Producer maximizes profit:

$$\boxed{\frac{P_X}{P_Y}} = MRT_{XY}$$

- Therefore:

$$MRS_{XY} \neq MRT_{XY}$$

- The resulting allocation is not Pareto optimal.

Key Insight

A tax drives a wedge between the marginal rate of substitution (consumers) and the marginal rate of transformation (producers), creating an excess burden (deadweight loss). The economy no longer satisfies the Pareto optimality condition $MRS = MRT$.

7.2 Optimal Taxation

7.2.1 Basic Concepts

- From an efficiency perspective, the lump-sum tax is optimal because it does not create substitution effects or excess burden.
- However, lump-sum taxes are unfair. From an equity perspective, individuals with higher ability should pay higher lump-sum taxes.

- But ability is unobservable. Therefore, in practice, governments must tax observable outcomes such as income or consumption.
- Taxing outcomes distorts prices (of goods or factors) and creates efficiency losses.
- Two main frameworks for optimal taxation:
  * Ramsey framework: Linear taxes $t \times x$
  * Mirrleesian framework: Nonlinear taxes $t(x)$

7.2.2 Ramsey Optimal Taxation: Model Setup

- The government imposes taxes to achieve two objectives:
  * Raise a fixed amount of revenue E
  * Minimize the utility loss of a representative consumer
- Lump-sum taxes are not available.
- N goods: Producer prices are fixed and normalized to 1.
  * Producer price for good i: $p_i = 1$
  * Consumer price for good i: $q_i = 1 + \tau_i$
- One representative consumer

The representative household maximizes:

$$\max_{x_1,\dots,x_n,l} U(x_1, x_2, \dots, x_n, l)$$

subject to the budget constraint:

$$q_1x_1 + q_2x_2 + \cdots + q_nx_n \leq wl + Z$$

where:

- Z: non-labor income
- w: wage rate
- Labor income is not taxed (if labor could be taxed, it would be equivalent to taxing all goods uniformly)

The Lagrangian is:

$$L = U(x_1, x_2, \dots, x_n, l) + \alpha(wl + Z - q_1x_1 - \cdots - q_nx_n)$$

First-order conditions:

$$MU_i = \alpha q_i, \quad MU_l = -\alpha w$$

The optimal choices $x_i(q, Z), l(q, Z)$ and the indirect utility function $V(q, Z)$ satisfy:

$$\frac{\partial V(q, Z)}{\partial Z} = \alpha, \quad \frac{\partial V(q, Z)}{\partial q_i} = -\alpha x_i$$

Proof for the Envelope Theorem

$$\frac{\partial V(q,Z)}{\partial Z} = \frac{\partial L}{\partial Z} = \sum_{i=1}^{n}(MU_i - \alpha q_i)\frac{\partial x_i}{\partial Z} + (MU_l - \alpha w)\frac{\partial l}{\partial Z} + \alpha = \alpha$$

$$\frac{\partial V(q,Z)}{\partial q_i} = \frac{\partial L}{\partial q_i} = \sum_{j=1}^{n}(MU_j - \alpha q_j)\frac{\partial x_j}{\partial q_i} + (MU_l - \alpha w)\frac{\partial l}{\partial q_i} - \alpha x_i = -\alpha x_i$$

α represents the marginal utility of income (marginal value of money for the individual), also known as the shadow price.

Roy's identity:

$$x_i = -\frac{\frac{\partial V(q,Z)}{\partial q_i}}{\frac{\partial V(q,Z)}{\partial Z}}$$

Government's Problem

The government chooses consumer prices $q_1, q_2, \ldots, q_n$ to maximize consumer utility subject to raising revenue E:

$$\max_q \; V(q,Z) \quad \text{s.t.} \quad \sum_{i=1}^{n} \tau_i x_i(q,Z) \geq E$$

Government's Lagrangian

$$\mathcal{L}_G = V(q,Z) + \lambda\left[\sum_{i=1}^{n} \tau_i x_i(q,Z) - E\right]$$

First-order condition for q_i:

$$\frac{\partial \mathcal{L}_G}{\partial q_i} = \frac{\partial V(q,Z)}{\partial q_i} + \lambda\left[x_i(q,Z) + \sum_j \tau_j \frac{\partial x_j(q,Z)}{\partial q_i}\right] = 0$$

- Effect on utility: $\frac{\partial V}{\partial q_i}$
- Effect on tax revenue: $x_i + \sum_j \tau_j \frac{\partial x_j}{\partial q_i}$

Interpretation of λ

$$\frac{\partial V(q,Z,E)}{\partial E} = \sum_{i=1}^{n} \frac{\partial \mathcal{L}_G}{\partial q_i}\frac{\partial q_i}{\partial E} + \frac{\partial \mathcal{L}_G}{\partial E} = \frac{\partial \mathcal{L}_G}{\partial E} - \lambda$$

- λ: Marginal social welfare loss from an additional unit of revenue requirement (increase in E reduces utility by λ)
- α: Marginal utility of private income (increase in Z increases utility by α)

Optimal Tax Conditions

Substituting $\frac{\partial V}{\partial q_i} = -\alpha x_i$ into the first-order condition:

$$(\lambda - \alpha)x_i + \lambda \sum_j \tau_j \frac{\partial x_j(q, Z)}{\partial q_i} = 0, \quad i = 1, 2, \ldots, n$$

This system of n equations determines the optimal tax rates $\tau_1, \ldots, \tau_n$.

Further Simplification: Defining θ

Definition of θ

$$\theta = (\lambda - \alpha) - \lambda \frac{\partial \sum_j \tau_j x_j}{\partial z}$$

θ represents: the value for the government of introducing a 1 unit lump-sum tax.

- λ: When tax revenue increases by 1 unit, consumer utility increases by λ.
- α: When disposable income decreases by 1 unit, consumer utility decreases by α.
- $\lambda \frac{\partial \sum_j \tau_j x_j}{\partial z}$: When disposable income decreases by 1 unit, the consumption bundle x_j changes, causing commodity tax revenue to change by $\frac{\partial \sum_j \tau_j x_j}{\partial z}$. The value of each unit of tax revenue is λ.

It can be shown that $\theta > 0$.

Starting from the first-order condition:

$$(\lambda - \alpha)x_i + \lambda \sum_j \tau_j \frac{\partial x_j(q, z)}{\partial q_i} = 0$$

Using the *Slutsky equation*:

$$\frac{\partial x_j}{\partial q_i} = \frac{\partial h_j}{\partial q_i} - x_i \frac{\partial x_j}{\partial z}$$

Substituting:

$$(\lambda - \alpha)x_i + \lambda \sum_j \tau_j \left(\frac{\partial h_j}{\partial q_i} - x_i \frac{\partial x_j}{\partial z} \right) = 0$$

Dividing by x_i and rearranging:

$$(\lambda - \alpha) + \lambda \sum_j \tau_j \frac{\partial h_j}{\partial q_i} \cdot \frac{1}{x_i} - \lambda \sum_j \tau_j \frac{\partial x_j}{\partial z} = 0$$

Using the definition of θ:

$$\sum_j \frac{\tau_j}{x_j} \frac{\partial h_j}{\partial q_i} = -\frac{\theta}{\lambda}$$

Optimal Taxation in Terms of Price Elasticities

Using symmetry of the compensated demand derivatives $\frac{\partial h_j}{\partial q_i} = \frac{\partial h_i}{\partial q_j}$ and noting that $h_i = x_i$ at the optimum:

$$\sum_j \frac{\tau_j}{x_i} \frac{\partial h_i}{\partial q_j} = -\frac{\theta}{\lambda}$$

Multiplying and dividing by q_j:

$$\sum_j \frac{\tau_j}{q_j} \cdot \frac{\partial h_i}{\partial q_j} \cdot \frac{q_j}{x_i} = -\frac{\theta}{\lambda}$$

Since $\frac{\tau_j}{q_j} = \frac{\tau_j}{1+\tau_j}$ (as $q_j = 1 + \tau_j$), we obtain:

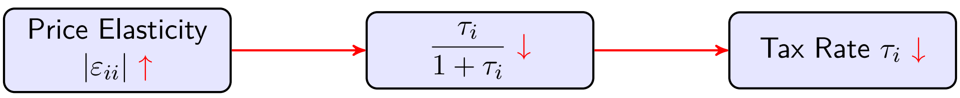
$$\sum_j \frac{\tau_j}{1+\tau_j} \varepsilon_{ij} = -\frac{\theta}{\lambda}$$

where ε_{ij} is the compensated cross-price elasticity of good i with respect to the price of good j:

$$\varepsilon_{ij} = \frac{\partial h_i}{\partial q_j} \cdot \frac{q_j}{x_i}$$

If cross-price elasticities are zero, i.e., $\varepsilon_{ij} = 0$ for $i \neq j$, then:

$$\frac{\tau_i}{1+\tau_i} = \frac{\theta}{\lambda} \cdot \frac{1}{|\varepsilon_{ii}|}$$



7.12: Relationship between Price Elasticity and Optimal Tax Rate

- Larger price elasticity $|\varepsilon_{ii}|$ $\rightarrow$ Smaller $\frac{\tau_i}{1+\tau_i}$ $\rightarrow$ Smaller tax rate τ_i
- This is the classic inverse elasticity rule in optimal taxation: goods with more elastic demand should be taxed at lower rates to minimize excess burden.

7.2.3 Limitations of the Inverse Elasticity Rule

- Inverse elasticity rule: Tax goods with inelastic demand at higher rates.
- Limitation: Only considers efficiency, ignores equity concerns.
- Compared to luxuries, necessities tend to have lower elasticities.
- Therefore, Ramsey optimal commodity taxes are likely to be regressive, meaning the poor face higher tax rates.
- Diamond (1975) extended the Ramsey model to incorporate distributional considerations

7.2.4 Special case: Two types of products

- Suppose there are only two goods, x_1 and x_2.
- Let leisure be denoted as good x_0. Including leisure, there are three goods in total.
- From previous derivations, the optimal tax rates satisfy:

$$\sum_j \frac{\tau_j}{1+\tau_j}\varepsilon_{ij} = -\frac{\theta}{\lambda}, \quad i = 1, 2$$

Thus:

$$\frac{\tau_1}{1+\tau_1}\varepsilon_{11} + \frac{\tau_2}{1+\tau_2}\varepsilon_{12} = -\frac{\theta}{\lambda}$$
$$\frac{\tau_1}{1+\tau_1}\varepsilon_{21} + \frac{\tau_2}{1+\tau_2}\varepsilon_{22} = -\frac{\theta}{\lambda}$$

Solving:

$$\frac{\tau_1}{1+\tau_1} = \frac{\theta}{S\lambda}(\varepsilon_{12} - \varepsilon_{22})$$
$$\frac{\tau_2}{1+\tau_2} = \frac{\theta}{S\lambda}(\varepsilon_{21} - \varepsilon_{11})$$

where $S = \varepsilon_{11}\varepsilon_{22} - \varepsilon_{12}\varepsilon_{21} > 0$.

From the properties of compensated demand elasticities:

$$\varepsilon_{11} + \varepsilon_{12} + \varepsilon_{10} = 0$$
$$\varepsilon_{21} + \varepsilon_{22} + \varepsilon_{20} = 0$$

Then, we can get:

$$\frac{\tau_1}{1+\tau_1} = \frac{\theta}{S\lambda}(\varepsilon_{12} + \varepsilon_{21} + \varepsilon_{20})$$

$$\frac{\tau_2}{1+\tau_2} = \frac{\theta}{S\lambda}(\varepsilon_{12} + \varepsilon_{21} + \varepsilon_{10})$$

- If $\varepsilon_{20} > \varepsilon_{10}$, then $\tau_1 > \tau_2$.
- $\varepsilon_{20} > \varepsilon_{10}$ implies that good 1 has a larger substitution effect with leisure compared to good 2.
- Tax goods that are complements to leisure more heavily.(Corlett and Hague, 1953)

If All Goods Can Be Taxed

Suppose leisure is not included in the utility function, i.e., there are only two goods, and both can be taxed.

$$\frac{\tau_1}{1+\tau_1} = \frac{\theta}{S\lambda}(\varepsilon_{12} - \varepsilon_{22})$$

$$\frac{\tau_2}{1+\tau_2} = \frac{\theta}{S\lambda}(\varepsilon_{21} - \varepsilon_{11})$$

From the properties of compensated elasticities:

$$\varepsilon_{11} + \varepsilon_{12} = 0, \quad \varepsilon_{21} + \varepsilon_{22} = 0$$

Thus:

$$\frac{\tau_1}{1+\tau_1} = \frac{\theta}{S\lambda}(\varepsilon_{12} + \varepsilon_{21})$$
$$\frac{\tau_2}{1+\tau_2} = \frac{\theta}{S\lambda}(\varepsilon_{12} + \varepsilon_{21})$$

- This implies $\tau_1 = \tau_2$.
- If all goods (including leisure) can be taxed, then uniform taxation is optimal.

Key Takeaways

Ramsey Optimal Commodity Taxation:

Model Setup

- **Producer:** n goods, producer prices fixed at 1 (perfectly elastic supply). If the tax on good i is τ_i, the consumer price is $q_i = 1 + \tau_i$.
- **Household:** One representative household with utility determined by consumption of n goods and labor supply. Given consumer prices $(q_1, q_2, \ldots, q_n)$, wage rate w, and non-labor income Z, the household chooses optimal labor supply and consumption bundle to maximize utility:

$$\max_{x_1,\ldots,x_n,l} \; U(x_1, x_2, \ldots, x_n, l)$$
$$\text{s.t.} \quad q_1 x_1 + q_2 x_2 + \cdots + q_n x_n \leq wl + Z$$

- **Government:** Taxes commodities to raise revenue E. The goal of optimal taxation is to maximize the utility of the representative household subject to the revenue constraint.

Basic Result

- Inverse Elasticity Rule: Tax goods with more inelastic demand at higher rates to minimize excess burden.

Appendix A Problem Set

Instructions

This problem set covers key topics in public economics and welfare theory, including the First Welfare Theorem, externalities, public goods, mechanism design, adverse selection, moral hazard, pension economics, tax incidence, and excess burden.

Problem 1: First Fundamental Theorem of Welfare Economics

In a pure exchange perfectly competitive market, there are two consumers, A and B, and two goods, X and Y. Initially, A owns 3 units of X and 8 units of Y, and B owns 1 unit of X and 8 units of Y. Their utility functions are: $U(X_A, Y_A) = X_A Y_A$ and $U(X_B, Y_B) = X_B Y_B$. Find:

1. The (relative) price and the consumption of each good for each person in the competitive market equilibrium.
2. The equation for the contract curve, which represents all Pareto optimal allocations. Does the competitive market equilibrium lie on the contract curve?
3. Is the competitive market equilibrium always Pareto optimal? Explain intuitively.

Problem 2: Externalities

In a perfectly competitive market, there is one representative consumer with utility $u(x; P) = \frac{1}{2}\ln x + y - \frac{3}{2}P$, where x represents the quantity of cars, y is consumption of all other goods, and P is the level of air pollution. The consumer's income is 100.

A car manufacturer has a cost function $c(x) = x^2$. Assume that producing one unit of car generates one unit of air pollution.

1. In a perfectly competitive market, what is the output of cars?
2. What is the socially optimal output of cars?
3. What per-unit tax on cars in the competitive market would achieve the socially optimal outcome?

Problem 3: Samuelson Rule and Lindahl Prices

Suppose there are two representative consumers, A and B, who consume only two goods, X and G. Their utility functions are quasi-linear: $u_A(X, G) = X + \ln G$, $u_B(X, G) = X + 2 \ln G$. The incomes of A and B are $I_A = 40$ and $I_B = 60$, respectively. The production possibility frontier is given by $X + 3G = 100$.

1. If both X and G are private goods, find the Pareto-efficient quantities of the two goods.
2. If X is a private good and G is a public good, find the Pareto-efficient quantities of the two goods.
3. If G is a public good, find the Lindahl equilibrium. Specifically, how much would A and B each pay per unit of the public good? What is the quantity of the public good provided? (Hint: In a perfectly competitive market, the price of X is 1, and the price of G is 3.)

Problem 4: Clarke Mechanism Design

Suppose a house has three residents: A, B, and C. The government is considering installing running water in the house. Assume the utilities derived from having running water are $u_A = 4000$, $u_B = 5000$, and $u_C = 11000$ (all measured in monetary units). The cost of installation is $c = 18000$, which is to be split equally among the three, so each pays 6000. The decision is made by asking A, B, and C to report their net benefit s_i. Running water is installed if $s_A + s_B + s_C > 0$.

1. From a social perspective, is installing running water worthwhile?
2. Given that B and C tell the truth, i.e., $s_B = -1000, s_C = 5000$, is it optimal for A to tell the truth or not? (Assume no Clarke tax is levied here.)
3. Now assume the Clarke tax mechanism is in place. Prove that:
   (a) If B and C tell the truth, telling the truth is (one of) A's optimal strategies.
   (b) If B and C lie (for example, $s_B = -500, s_C = 1000$), telling the truth is still (one of) A's optimal strategies.

Problem 5: Health Insurance and Adverse Selection

Assume there are two representative consumers, L and H. Both have the utility function $U(C) = \sqrt{C}$, where C is consumption net of medical expenses. Their initial wealth is 3600. L is a low-risk individual with a 20% probability of illness. H is a high-risk individual with a 30% probability of illness. If ill, medical expenses are 1100. Each person knows their own probability of illness.

1. Assuming L and H can both buy full health insurance, what is the maximum amount L is willing to pay for this insurance? What is the maximum amount H is willing to pay?

2. Under perfect information, the insurance company can distinguish between low-risk and high-risk individuals. What are the fair premiums for L and H? Who will buy insurance?
3. Under asymmetric information, the insurance company cannot distinguish between risk types but knows the proportion of low-risk individuals in the market.
   (a) Suppose the market consists of 50% low-risk and 50% high-risk individuals. If everyone buys insurance, what is the fair premium? Knowing this premium, will both L and H buy insurance?
   (b) Suppose the market consists of 90% low-risk and 10% high-risk individuals. If everyone buys insurance, what is the fair premium? Knowing this premium, will both L and H buy insurance?
4. Discuss the rationale for social insurance from the perspective of information asymmetry.

Problem 6: Health Insurance and Moral Hazard

A representative consumer has utility function $U(C) = \sqrt{C}$, where C is consumption net of medical expenses. Their income is 6400. Without preventive measures, there is a 0.7 probability of illness, which would result in medical expenses of 4800. The household can take preventive measures (e.g., quitting smoking, exercising more) at a cost of 1500, which reduces the probability of illness to 0.2. The household can choose whether to purchase full health insurance. Assume whether preventive measures are taken is private information unobservable to the insurance company.

1. Without insurance, what is the expected utility if the household takes preventive measures? What is the expected utility if it does not? Will the household take preventive measures?
2. Suppose the household can now purchase full health insurance. Based on past experience, the insurance company believes the household is low-risk and charges a fair premium based on the low probability of illness. Will the household buy insurance? After buying insurance, will it take preventive measures?
3. Knowing the household's behavior after purchasing insurance, will the insurance company still charge the fair premium based on the low-risk probability? If not, what is the new fair premium? Will the household still buy insurance?
4. What measures can the insurance company take to mitigate moral hazard?

Problem 7: Pension Economics and Labor Supply

In a two-period model, a myopic consumer only cares about first-period utility. Their utility function is $u(c) + h(1 - l)$, where c and l represent consumption and labor supply, respectively. The wage per unit of labor is w, and the pension contribution rate is τ.

1. Suppose the utility functions take the form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $h(1 - l) = 1 - l$. How does the pension system affect labor supply?
2. Explain intuitively the two effects of the pension system on labor supply.

Problem 8: Tax Incidence in Partial Equilibrium (1)

In a perfectly competitive market, the demand function for a good is $Q = 4000 - 100P$, and the supply function is $Q = -200 + 100P$.

1. Find the equilibrium price and quantity.
2. If a \$2 excise tax is imposed on this good, how much of the tax burden is borne by producers and how much by consumers?
3. Who bears more of the tax burden, consumers or producers? Why?

Problem 9: Tax Incidence in Partial Equilibrium (2)

In a perfectly competitive market, the demand function for a good is $Q = a - bP$, and the supply function is $Q = c + dP$, with $b, d > 0$. Assume $b > d$.

1. Find the equilibrium price and quantity.
2. At the equilibrium price and quantity, which is larger, the price elasticity of demand or the price elasticity of supply?
3. If a \$$t$ excise tax is imposed on this good, how much of the tax burden is borne by consumers and how much by producers?
4. Who bears more of the tax burden? Provide an intuitive explanation.
5. If $d = 0$, what is the tax incidence? Provide an intuitive explanation.

Problem 10: Tax Incidence in General Equilibrium

In a perfectly competitive market with two goods, the production function for good X is $X = K^{\frac{1}{3}}L^{\frac{2}{3}}$, and the production function for good Y is $Y = K^{\frac{2}{3}}L^{\frac{1}{3}}$. The representative household owns 1 unit of labor and 1 unit of capital, and has utility function $U(X, Y) = XY$.

1. Find the equilibrium factor prices and product prices.
2. If the government imposes a 50% ad valorem tax (exclusive tax) on good Y and returns all tax revenue to the representative household as a lump-sum transfer, what are the new equilibrium factor prices and product prices?
3. Explain intuitively the changes in factor prices after the tax is imposed.

Problem 11: Measuring Excess Burden with Equivalent and Compensating Variation

Suppose a consumer consumes only two goods with utility function $u(x, y) = xy$. Market prices are $P = (2, 1)$, i.e., the price of X is 2 and the price of Y is 1. The consumer's income is $m = 80$. Assume supply elasticity is infinite, so the full tax burden is passed on to consumers.

1. Suppose the government imposes a \$1 per-unit tax on good Y. Calculate the excess burden of the tax using both equivalent variation and compensating variation.
2. Suppose the government imposes a \$2 per-unit tax on good X and a \$1 per-unit tax on good Y simultaneously. Calculate the excess burden of this tax package using both equivalent variation and compensating variation.